Modeling non-trading days in risk forecasting

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When forecasting daily market risk, a public holiday's zero return leads to a lower, and distorted, risk estimate. We tested different methods for imputing holiday returns and analyzed whether they smooth risk forecasts and reduce turnover in DPPI risk budgeting strategies.

Forecasting daily market risk involves several practical difficulties. For example: public or bank holidays, when exchanges are closed and prices do not change. To account for these, some risk models may assume a daily return of zero – which can potentially have a significant impact on the model output. In Copula-GARCH models, for example, which assign a significant weight to the most recent data, the zero-return assumption will result in a lower risk estimate.

For a risk budgeting strategy like Dynamic Proportion Portfolio Insurance (DPPI), a lower risk estimate can lead to a higher target exposure, potentially inducing a buy trade. This means that, when the market reopens, the price may rise disproportionately, leading to a higher risk estimate and a lower target exposure – and a sell trade. Thus, inadequate modeling of non-trading days may generate unnecessary turnover, and this effect is particularly pronounced when market risk is already high and risk management is at the forefront.

There are different ways to avoid this kind of artificial back and forth: An intuitive and simple method would be to copy forward the risk estimate rather than the last price. But such an approach disregards what happens in the other (open) markets. In periods of high volatility, investors would prefer the risk forecast to increase rather than to remain constant. In this article, we will assess various methods that can tackle this problem.

Imputing returns of non-trading days

Forecasting returns, particularly daily returns, is extremely difficult (Rapach and Zhou, 2013). Fortunately, we are not interested in the exact return, but only in its magnitude. This will be the main driver

About risk: The value of investments and any income will fluctuate (this may partly be the result of exchange rate fluctuations) and investors may not get back the full amount invested.

Risk modeling

Modern risk modeling is guided by empirical patterns, which cannot be adequately captured with a conventional normal distribution assumption. Extreme events occur far more often than the normal distribution suggests. Volatility and correlations are not constant, and volatility clustering is not uncommon.

An effective method of understanding empirical risk is the Copula-GARCH model, as proposed by Patton (2006) or Jondeau and Rockinger (2006): In the first step, risk dynamics are measured by fitting univariate GARCH(1,1) models to the underlying return series. Assuming a return process $(r_{i,t})_{i \in N, t \in Z}$, the mean and variance equations are given by:

$$r_{i,t} = \mu_i + \varepsilon_{i,t}$$

$$\varepsilon_{i,t} = z_{i,t}\sqrt{\sigma^2_{i,t}}$$

$$z_{i,t} \sim D_i(0, 1, \xi_i, \upsilon_i)$$

$$\sigma^2_{i,t} = \omega_i + \alpha_i \varepsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1}$$

where $\omega_i > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$, $i = 1,\ldots,N$. Moreover, $r_{i,t}$ are the returns of the i^{th} portfolio asset at time t , and D_i reflects the skewed t-distribution with skewness parameter ξ_i and shape parameter υ_i according to Hansen (1994).

In the second step, a time-varying copula permits us to estimate the marginal distributions of the asset returns together with the dependence structure. In particular, the joint distribution of the *N*GARCH return processes can be expressed depending on an *N*-dimensional copula *C*:

$$F_t(r_t | \mu_t, \sigma_t) = C_t(F_{1,t}(r_{1,t} | \mu_{1,t}, \sigma_{1,t}),\ldots, F_{N,t}(r_{N,t} | \mu_{N,t}, \sigma_{N,t}) | F_{t-1})$$

where $F_1(\bullet),\ldots,F_N(\bullet)$ are the conditional marginal distributions of the return processes. The dependence structure of the margins is assumed to follow a Student's *t*-copula with conditional correlation R_t and constant shape parameter η. We opt for the Student's *t*-copula for modeling the dependence of financial assets, since the normal copula cannot account for tail dependence. The conditional density of the Student's *t*-copula at time *t* is given by:

$$c_t(u_{i,t},\ldots,u_{N,t} | R_t, \eta) = \frac{f_t\left(F^{-1}_{i,t}(u_{i,t}|\eta),\ldots,F^{-1}_{i,t}(u_{N,t}|\eta) | R_t, \eta\right)}{\prod^n_{i=1} f_i\left(F^{-1}_{i,t}(u_{i,t}|\eta)|\eta\right)}$$

where $u_{i,t} = F_{i,t}(r_{i,t} | \mu_{i,t}, \sigma_{i,t}, \xi_i, \nu_i)$ is the probability integral transformation of each series by its conditional distribution $F_{i,t}$ estimated via the first-stage GARCH process, $F^{-1}_{i,t}(u_{i,t}|\eta)$ represents the quantile transformation of the uniform margins subject to the common shape parameter of the multivariate density, $F_t(\bullet | R_t, \eta)$ is the multivariate density of the Student's t-distribution with conditional correlation R_t and shape parameter η and $f_i(\bullet | \eta)$ defines the univariate margins of the multivariate Student's *t*-distribution with common shape parameter η. Furthermore, we allow the parameters of the conditional copula to vary with time in a manner analogous to a GARCH model for conditional variance (e.g., Patton, 2006). Specifically, we assume the dynamics of R_t to follow an asymmetric generalized dynamic conditional correlation (AGDCC) model according to Cappiello, Engle and Sheppard (2006).

Based on the copula estimates, we then generate *N* sets of random pseudo-uniform variables and transform these into corresponding realizations of the error processes by using the quantile function of the margins. These simulated numbers are then used together with the conditional volatility forecast of the GARCH models to derive a Monte Carlo set of returns for each asset.[1]

Another matter to consider, in addition to the structure of the model itself, is that of an appropriate risk measure. Whereas many risk management approaches rely on value-at-risk (VaR), risk budgeting strategies naturally lend themselves to using expected shortfall (ES) to measure risk. In the case of VaR, it indicates the maximum possible loss at a given confidence level (usually 95% or 99%). However, VaR is silent with respect to the losses beyond the VaR threshold. Conversely, ES measures the expected loss in the event of a VaR violation. Hence, by means of the portfolio's weight vector, we can then compute a distribution of portfolio returns for t+1 which allows us to calculate VaR and ES forecasts.

of the final risk forecast, in particular since, in the GARCH model, the return is squared. Given the stylized facts of financial asset returns, such as volatility clustering and correlations between related markets, we opt for the following methodologies to generate the imputed return $\hat{r}_t$:

1. **Simple average:**

   $$\hat{r}_t = \frac{\sum^{t=-20}_{t=-1} r_t}{20}$$

   or the average return over the last 20 days (approximately one month of trading returns).

2. **Last day:** $\hat{r}_t = r_{t-1}$
   Here we assume that the best prediction for the magnitude of the next day's market return is simply the magnitude of the current return. This could be an alternative in the case of volatility clustering.

3. **Cross market:**

   $$\hat{r}_t = \frac{\sum^{i=n}_{i=1} r_{i_{t-1}}}{n}$$

   where *i* is a related market (e.g., same asset class) and *n* is the number of related markets. With this approach, we aim to capture information from open markets in a simple manner.

4. **VaR model:** $\hat{r}_t = v + A_1 * r_{t-1}$,
   where $\hat{r}_t$ is the vector of returns to impute, *v* and A_1 are the model coefficients and r_{t-1} gives us the returns from the previous period. We use 500 days of lagged returns to estimate the model coefficients.

5. **Linear regression model on open markets (Linear model):**

   $$\hat{r}_t = a + \sum^{n}_{i=1} b_i * r_{i,t},$$

   where $\hat{r}_t$ is the return to impute, *i* is a related open market, *n* is the total number of related markets, b_i reflects the coefficients with respect to related open markets and $r_{i,t}$ is the return of the open market for the same time period.

6. **Enhanced linear regression model (Enhanced LM):**
   The enhanced linear regression model follows the same logic as method 5, but attempts to capture autocorrelation and volatility clustering by including 20 lags of the same time series. The equation is as follows:

$$\widehat{r_t} = a + \sum_{i=1}^{n} b_i * r_{i,t} + \sum_{k=1}^{k=20} c_k * r_{j,t-k},$$

   where c_k reflects the coefficients with respect to the own lagged series j.

Finally, suppose a market was closed from Monday through Thursday – the Friday return will likely be very high (or low), since it reflects the information of the whole week (figure 1).

For this reason, we adjust the realized return after the market reopens using the imputed returns of the prior days, as in the following equation, and apply the adjusted return $\widehat{R_5}$:

$$\widehat{R_5} = \frac{1 + R_5}{(1+\widehat{R_1}) * (1+\widehat{R_2}) * (1+\widehat{R_3}) * (1+\widehat{R_4})} - 1$$

Forecasting capability

To assess the forecasting capability of the different methods using daily return data from March 20, 2001 to February 6, 2023, we first look at the methods' general forecasting power: We impute returns for all days (except for an initial estimation window) and then compare the imputed to the realized returns using the mean squared prediction error (MSPE) of each method. This will not include non-trading days (as no realized returns are observed on these days), but rather provides information on which method generally works best for predicting returns.

Table 1 shows the mean squared prediction errors for an asset universe of stock indices, government bonds, credits, commodities and foreign exchange. For each asset, the method with the lowest MSPE is shown in boldface. The Enhanced Linear Model delivers the smallest prediction errors in all but two cases, with US and Euro investment grade bonds the only exceptions.

To assess whether the differences in MSPEs between different methods are statistically significant, we perform modified Diebold-Mariano tests (Diebold and Mariano, 1995; Harvey, Leybourne and Newbold, 1997). In these tests, each model is tested against each other model to determine which of the pair has the better forecasting accuracy. Table 2 shows the p-values for the S&P 500. Again, Enhanced LM is best, providing better forecasts than each of the other four models. Then follows (in order) the Linear model, Cross market, Simple average and the VaR model. "Last day" comes in last.

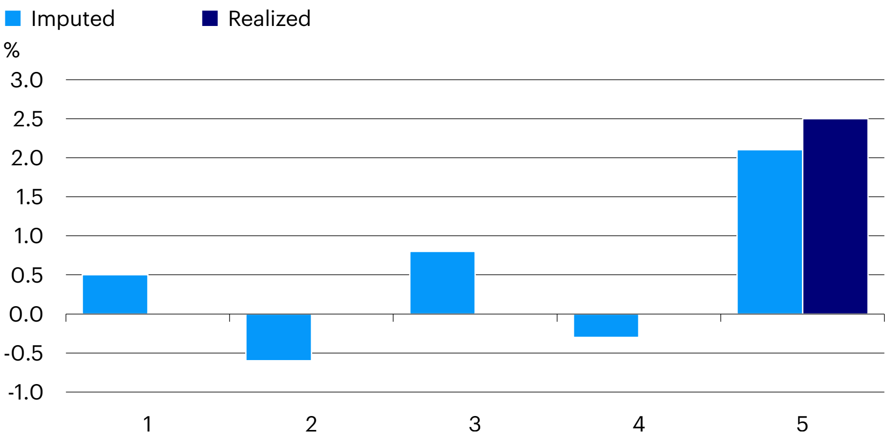
In a second step, we repeat the analysis for the days with the most extreme market movements (see table 3). Getting these right is of particular importance. Again, the Enhanced Linear Model performs best in all but two cases, which is confirmed by the Diebold-Mariano tests, with the other models following in the same order as in the full dataset case.

Expected shortfall

Using the Copula-GARCH model, we now compute expected shortfall (ES) forecasts for the S&P 500 as well as a multi-asset portfolio consisting of equity indices, government bonds, credits and commodities.[2] We analyze all available triplets of ES forecasts for the day before the non-trading day, the non-trading day itself and the day after – 151 triplets altogether.

In figure 2, panel A shows the mean of all 151 forecast triplets for the S&P 500. We see a pronounced V-shape for the no adjustment case, and less pronounced V-shapes for some of our six forecast models. Only the "Last day" method is clearly off: It's risk forecasts for the day after the non-trading day are much too high. These findings are supported by the results for the multi-asset portfolio in panel B. In the no adjustment case, the V-shape is even more pronounced, stressing the need for an adjustment of some sort.

Figure 1
Adjustment of a daily return after four consecutive days of the market being closed



Source: Invesco. For illustrative purposes only.

Table 1
General forecasting power of the models

Mean squared prediction errors (MSPE)		Simple average	Last day	Cross market	VaR model	Linear model	Enhanced LM
Stocks	S&P500	1.18%	1.78%	1.07%	1.31%	0.92%	**0.82%**
	EUROSTOXX50	1.41%	2.09%	0.99%	1.56%	0.69%	**0.66%**
	FTSE100	1.12%	1.64%	0.74%	1.23%	0.55%	**0.52%**
	MSCI EM	1.13%	1.48%	0.86%	1.04%	0.79%	**0.74%**
	TOPIX	1.35%	1.99%	1.38%	1.62%	1.14%	**1.09%**
Government bonds	AUS10Y	0.43%	0.64%	0.45%	0.55%	0.39%	**0.38%**
	CAN10Y	0.35%	0.50%	0.26%	0.36%	0.19%	**0.18%**
	US10Y	0.36%	0.52%	0.28%	0.38%	0.20%	**0.19%**
	JGB10Y	0.17%	0.26%	0.30%	0.20%	0.16%	**0.16%**
	UK10Y	0.40%	0.58%	0.30%	0.41%	0.24%	**0.22%**
	Euro Bund	0.34%	0.49%	0.24%	0.35%	0.19%	**0.18%**
Credits	EM sovereigns	0.47%	0.64%	0.44%	0.43%	0.39%	**0.36%**
	US IG	0.15%	0.16%	0.20%	**0.09%**	0.11%	0.09%
	US HY	0.40%	0.51%	0.33%	0.33%	0.29%	**0.27%**
	Euro IG	0.11%	0.14%	0.29%	**0.09%**	0.10%	0.09%
	Euro HY	0.36%	0.41%	0.33%	0.26%	0.28%	**0.24%**
Commodities	Agriculture	1.11%	1.60%	1.34%	1.13%	1.01%	**0.98%**
	Copper	1.61%	2.41%	1.54%	1.77%	1.41%	**1.36%**
	Oil	2.49%	3.67%	2.36%	2.65%	2.33%	**2.13%**
	Gold	1.06%	1.54%	1.44%	1.09%	1.00%	**0.96%**
Currencies	USDEUR	0.56%	0.82%	0.47%	0.59%	0.25%	**0.24%**
	GBPEUR	0.48%	0.69%	0.43%	0.49%	0.41%	**0.39%**
	JPYEUR	0.67%	0.98%	0.67%	0.71%	0.49%	**0.48%**
	AUDEUR	0.64%	0.93%	0.54%	0.68%	0.37%	**0.35%**
	NZDEUR	0.65%	0.94%	0.56%	0.69%	0.41%	**0.40%**
	CADEUR	0.56%	0.81%	0.44%	0.59%	0.38%	**0.37%**
	CHFEUR	0.44%	0.64%	0.52%	0.49%	0.41%	**0.38%**
	NOKEUR	0.50%	0.74%	0.51%	0.54%	0.40%	**0.38%**
	SEKEUR	0.42%	0.62%	0.47%	0.46%	0.36%	**0.34%**
	DKKEUR	0.02%	0.04%	0.33%	0.03%	0.02%	**0.02%**
	EMEUR	0.54%	0.84%	0.42%	0.64%	0.27%	**0.25%**

Source: Invesco calculations. Daily data from March 20, 2001 to February 6, 2023. In each row, the smallest value is in bold, indicating the best forecasting power.

Table 2
P-values of Diebold-Mariano tests

P-values	Simple average	Last day	Cross market	VaR model	Enhanced LM	Linear model
Simple average		0	0.9999185	0.00E+00	1	1.00E+00
Last day	1.00E+00		1	1.00E+00	1	1.00E+00
Cross market	8.15E-05	0		1.32E-12	1	1.00E+00
VAR model	1.00E+00	0	1		1	1.00E+00
Enhanced LM	0.00E+00	0	0	0.00E+00		9.16E-11
Linear model	0.00E+00	0	0	0.00E+00	1	

Source: Invesco calculations. The table should be read row-wise: for instance, "Simple average" delivers better forecasts than "Last day", with a p-value of effectively 0 and worse forecasts than "Cross Market", since the p-value approaches 1.

In both panels – and particularly panel B – risk forecasts in the no adjustment case fluctuate considerably, which is mitigated by most of the six methods. This fluctuation is also visible in table 4, which shows the changes of the ES forecasts on the days before and after the non-trading day (first two columns) and the effect of the forecasting models (final two columns). Except for the "Last day" methodology in the S&P 500 case, the models lead to lower ES forecasts. They are particularly pronounced in the multi-asset case (see table 5).

The effects of our forecasting methodologies on a DPPI strategy

We now analyze the effect of our forecasting methodologies on a DPPI risk budgeting strategy. We assume a risk-averse investor who wants to limit portfolio drawdowns. In this approach, a certain drawdown limit is defined, which should not be exceeded in a specified period, typically a calendar year.

The target exposure depends not only on the risk forecast, but also on the available cushion C_t at time t. The cushion is the

Table 3
Forecasting power of the models for extreme market movements (1% quantile)

Mean squared prediction errors (MSPE)		Simple average	Last day	Cross market	VaR model	Linear model	Enhanced LM
Stocks	S&P500	4.39%	5.52%	3.39%	5.60%	3.26%	**2.85%**
	EUROSTOXX50	5.25%	6.50%	3.11%	6.06%	1.65%	**1.44%**
	FTSE100	4.63%	5.32%	2.05%	5.70%	1.71%	**1.42%**
	MSCI EM	5.23%	5.90%	2.12%	4.60%	2.16%	**1.77%**
	TOPIX	6.09%	5.84%	3.78%	6.29%	3.39%	**3.13%**
Government bonds	AUS10Y	1.48%	1.80%	1.56%	1.92%	1.25%	**1.10%**
	CAN10Y	1.29%	1.79%	0.80%	1.41%	0.45%	**0.47%**
	US10Y	1.38%	1.27%	1.01%	1.55%	0.70%	**0.61%**
	JGB10Y	0.66%	0.78%	0.70%	0.90%	0.66%	**0.57%**
	UK10Y	2.38%	3.06%	2.04%	2.22%	1.62%	**1.40%**
	Euro Bund	1.27%	1.53%	0.85%	1.33%	0.63%	**0.56%**
Credits	EM sovereigns	2.22%	2.53%	1.86%	1.94%	1.21%	**1.13%**
	US IG	0.54%	0.28%	0.19%	0.24%	0.21%	**0.18%**
	US HY	1.79%	1.37%	0.93%	1.46%	0.88%	**0.74%**
	Euro IG	0.39%	0.37%	0.64%	0.30%	0.37%	**0.33%**
	Euro HY	1.80%	1.18%	1.31%	1.46%	1.43%	**1.15%**
Commodities	Agriculture	3.75%	4.47%	**2.80%**	4.15%	2.97%	2.83%
	Copper	5.99%	8.49%	4.42%	7.39%	4.52%	**3.86%**
	Oil	13.80%	15.49%	12.97%	14.10%	12.43%	**9.67%**
	Gold	4.37%	5.40%	3.47%	4.04%	2.97%	**2.76%**
Currencies	USDEUR	2.26%	2.22%	1.61%	2.18%	0.54%	**0.40%**
	GBPEUR	1.82%	1.78%	1.34%	1.86%	1.33%	**1.07%**
	JPYEUR	2.61%	3.58%	2.86%	3.06%	1.03%	**0.93%**
	AUDEUR	3.15%	4.52%	2.73%	3.40%	1.16%	**1.04%**
	NZDEUR	2.69%	3.36%	2.16%	2.95%	0.97%	**0.92%**
	CADEUR	2.12%	2.56%	1.17%	2.12%	**0.67%**	0.71%
	CHFEUR	1.33%	1.31%	1.50%	1.45%	1.02%	**0.92%**
	NOKEUR	2.34%	2.06%	2.07%	2.49%	1.47%	**1.13%**
	SEKEUR	1.51%	2.00%	1.40%	1.71%	1.20%	**0.99%**
	DKKEUR	0.08%	0.14%	1.51%	0.11%	0.07%	**0.03%**
	EMEUR	2.10%	3.22%	1.21%	2.12%	0.73%	**0.55%**

Source: Invesco calculations. Daily data from March 20, 2001 to February 6, 2023. In each row, the smallest value is in bold, indicating the best forecasting power.

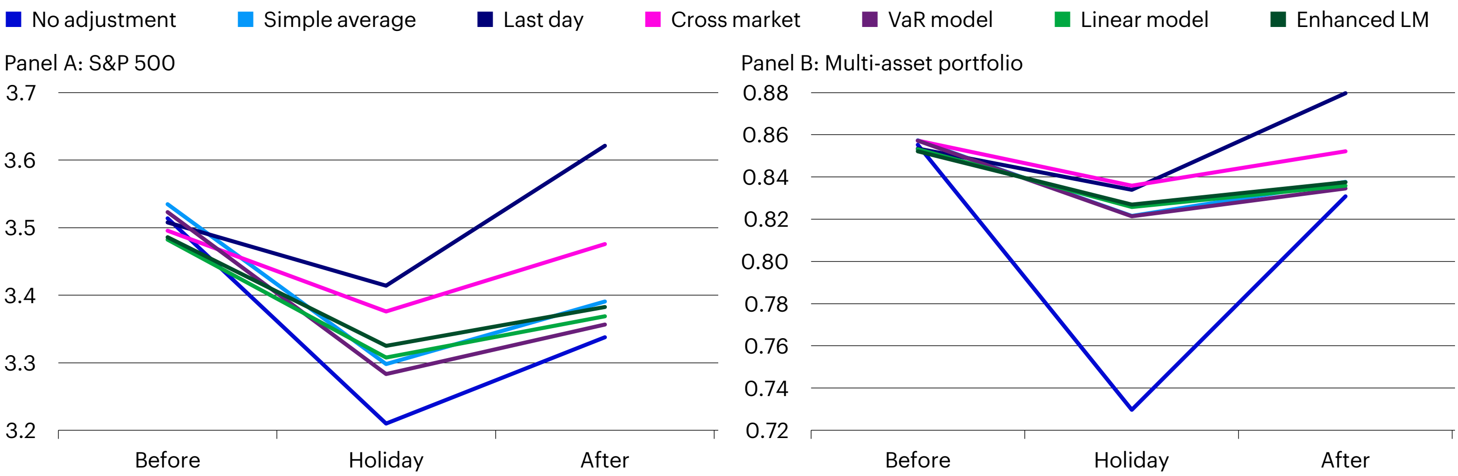
difference between the invested capital (W_t) and the net present value of the floor (F_t):

$$C_t = W_t - NPV(F_t)$$

To avoid losses in excess of the floor over the predefined time period, the target exposure e_t is a function of both the risk forecast and the available cushion at time t (C_t):

$$e_t = m_t * C_t,$$

Figure 2
Average ES forecasts for the 151 daily return triplets in our sample



Source: Invesco calculations. Daily data from March 20, 2001 to February 6, 2023.

Table 4
Fluctuations of average ES forecasts and forecasting model effects in the S&P 500 case

	Change of ES forecast since the previous day		Reduction of ES forecast due to the forecasting model	
	Day before	Day after	Day before	Day after
No adjustment	-7.29%	4.81%	-	-
Simple average	-6.15%	4.57%	-15.76%	-4.84%
Last day	-2.67%	7.79%	-63.44%	**62.12%**
Cross market	-2.67%	4.45%	-63.39%	-7.39%
VaR model	-6.24%	3.94%	-14.49%	-18.07%
Linear model	-4.44%	3.54%	-39.11%	-26.29%
Enhanced LM	-3.96%	3.23%	-45.72%	-32.77%

Source: Invesco calculations. Daily data from March 20, 2001 to February 6, 2023.

Table 5
Fluctuations of average ES forecasts and forecasting model effects in the multi-asset case

	Change of ES forecast since the previous day		Reduction of ES forecast due to the forecasting model	
	Day before	Day after	Day before	Day after
No adjustment	-13.45%	13.87%	-	-
Simple average	-3.41%	2.48%	-74.66%	-82.10%
Last day	-2.04%	5.93%	-84.80%	-57.24%
Cross market	-1.80%	2.38%	-86.59%	-82.83%
VaR model	-3.41%	2.09%	-74.64%	-84.94%
Linear model	-2.50%	1.80%	-81.38%	-87.03%
Enhanced LM	-2.27%	1.81%	-83.15%	-86.96%

Source: Invesco calculations. Daily data from March 20, 2001 to February 6, 2023.

The multiplier m_t is dynamic and a function of the risk forecast:

$$m_t = \frac{1}{\hat{\rho}_t MDD}$$

where $\hat{\rho}_t$ is the expected shortfall forecast at time t. Max drawdown days (MDD) is a risk aversion parameter, typically taking values between 1 and 5, which can be thought of as a linear extension of the number of days over which the drawdown can be suffered.

Tables 6 and 7 show the effect of our forecasting methodologies on the turnover of DPPI strategies, for both the S&P 500 case and the multi-asset case, for annual risk budgets from 1% to 10%. The turnover of an S&P 500 portfolio can be reduced by 18.48% (on average) in the case of the

Table 6
Turnover and turnover reduction for a DPPI strategy with different risk budgets: S&P 500 case

	Risk budget p.a.									
Turnover	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
No adjustment	8.49%	8.49%	8.49%	8.47%	7.81%	6.58%	5.87%	5.18%	4.48%	3.86%
Simple average	7.52%	7.52%	7.52%	7.50%	6.88%	5.73%	5.07%	4.42%	3.78%	3.32%
Last day	7.62%	7.62%	7.62%	7.59%	6.92%	5.81%	5.16%	4.41%	3.89%	3.28%
Cross market	7.56%	7.56%	7.56%	7.54%	6.89%	5.78%	5.06%	4.43%	3.79%	3.28%
VaR model	7.42%	7.42%	7.42%	7.39%	6.74%	5.53%	4.89%	4.26%	3.62%	3.12%
Linear model	7.11%	7.11%	7.11%	7.08%	6.40%	5.38%	4.77%	4.20%	3.56%	3.09%
Enhanced LM	7.03%	7.03%	7.03%	7.00%	6.36%	5.35%	4.75%	4.18%	3.56%	3.10%

	Risk budget p.a.										
Turnover reduction	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**	**Average**
No adjustment	-	-	-	-	-	-	-	-	-	-	-
Simple average	11.43%	11.43%	11.43%	11.42%	11.95%	12.98%	13.61%	14.60%	15.56%	14.18%	12.86%
Last day	10.25%	10.25%	10.25%	10.39%	11.35%	11.70%	12.20%	14.80%	13.16%	15.22%	11.96%
Cross market	10.94%	10.94%	10.94%	10.99%	11.83%	12.20%	13.93%	14.46%	15.47%	15.08%	12.68%
VaR model	12.65%	12.65%	12.65%	12.69%	13.73%	16.02%	16.82%	17.72%	19.16%	19.26%	15.34%
Linear model	16.27%	16.27%	16.27%	16.42%	18.09%	18.25%	18.86%	18.96%	20.42%	19.97%	17.98%
Enhanced LM	17.20%	17.20%	17.20%	17.33%	18.61%	18.69%	19.16%	19.34%	20.43%	19.67%	18.48%

Source: Invesco calculations. For illustrative purposes only.

Table 7
Turnover and turnover reduction for a DPPI strategy with different risk budgets: multi-asset case

	Risk budget p.a.				
Turnover	**1%**	**2%**	**3%**	**4%**	**5%**
No adjustment	13.78%	7.81%	3.32%	1.26%	0.51%
Simple average	4.81%	3.32%	1.57%	0.66%	0.34%
Last day	4.96%	3.31%	1.61%	0.74%	0.43%
Cross market	4.84%	3.46%	1.73%	0.85%	0.37%
VaR model	4.55%	3.19%	1.50%	0.61%	0.30%
Linear model	4.56%	3.19%	1.51%	0.57%	0.25%
Enhanced LM	4.56%	3.21%	1.48%	0.53%	0.24%

	Risk budget p.a.					
Turnover reduction	**1%**	**2%**	**3%**	**4%**	**5%**	**Average**
No adjustment	-	-	-	-	-	-
Simple average	65.07%	57.53%	52.79%	47.43%	33.62%	51.29%
Last day	64.00%	57.60%	51.57%	41.19%	15.86%	46.04%
Cross market	64.85%	55.64%	47.81%	32.38%	27.87%	45.71%
VaR model	66.99%	59.19%	54.86%	51.27%	40.24%	54.51%
Linear model	66.89%	59.13%	54.52%	54.45%	51.32%	57.26%
Enhanced LM	66.88%	58.84%	55.44%	58.10%	52.10%	58.27%

Source: Invesco calculations. For illustrative purposes only.

Enhanced LM methodology – but even "Last day" achieves an average reduction of 11.96%. In the multi-asset case, turnover reductions are also sizeable, with averages of up to 58.27%. Once again, the best result is achieved with the Enhanced LM methodology.

Conclusion

Not adjusting for non-trading days leads to higher risk forecast fluctuations and a higher portfolio turnover. We have tested different approaches for imputing non-trading day returns with the objective of ameliorating these problems. In most cases, all six methodologies deliver an improvement. Still, in our view, the Enhanced linear regression model (Enhanced LM) is the most appropriate choice given that it outperforms the other methods using a diverse set of evaluation metrics.

Notes

1 See Happersberger, Lohre and Nolte (2020) for further details on the applied risk model.
2 The multi-asset portfolio consist of 60% government bonds (German, UK, US, Canadian, Australian and Japanese; 10% each); 22% equities (S&P 500, EuroStoxx50, FTSE 100 and Topix; capitalization weighted), 10% commodities (2.5% oil, 5% gold, 2.5% copper), and 8% money market investments with practically no expected shortfall risk.

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