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# Bitcoin and Portfolio Choice: An Assessment of Bitcoin in Multi-Asset Portfolios

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## KEY FINDINGS

- Small allocations to BTC appear to offer notable benefits to multi-asset portfolios—even when return expectations for BTC are significantly reduced.
- Allocations to BTC should be evaluated relative to an investor’s initial risk preferences.
- BTC allocation sizing and portfolio rebalancing are critical aspects of risk management for portfolios with BTC allocations.

## ABSTRACT

In this article, the authors explore the implications for multi-asset investors of including an allocation to Bitcoin (BTC) in their portfolios. The authors examine the historical record to assess the observed characteristics of BTC and the historical performance of a broad range of stock/bond portfolios with various allocations to BTC, and they explore allocation implications across a wide range of simulated portfolio outcomes. The analyses provide insights into the benefits offered by small allocations to BTC, the importance of evaluating BTC allocations relative to an investor’s existing risk preferences, and the critical role of BTC allocation sizing and portfolio rebalancing for risk management.

In October 2008, Satoshi Nakamoto, the pseudonym for the still unverified author (or authors) of a white paper titled “Bitcoin: A Peer-to-Peer Electronic Cash System,” introduced Bitcoin (BTC) to the world. Originally conceived as an electronic cash system that was to serve as a replacement to the traditional banking system, the instrument was implemented in January of 2009. The intended use case as a store of value has since been limited by the extreme volatility exhibited over its short history. Currently, the question of whether BTC should be considered a currency, a commodity, or a collectible remains hotly debated among academics, investors, and even regulators. For example, Damodaran (2017) suggests that a possible path for BTC would be that it might be viewed as “gold for millennials.” More recently, Damodaran stated that “Bitcoin is the currency that nobody uses and a collectible that doesn’t behave like a collectible” (Business Today 2023). In short, clarity on the matter is still lacking, even among well-respected researchers. Other questions, such as how to value BTC or how to forecast BTC returns, also remain open.

Despite the open questions, the potential for the outsized returns offered by BTC have led many to begin considering it for inclusion in their portfolios. Perspectives on whether BTC should be considered as an investment generally range from deep skepticism to unbounded optimism, with seemingly little middle ground.<sup>1</sup> However, any objective assessment of BTC as an investment should include the following salient points:

1. BTC has exhibited substantially outsized returns relative to traditional asset classes.
2. BTC is a highly volatility instrument that exposes investors to remarkable run-ups and significant drawdowns.
3. The drivers of BTC return, risk, and correlation characteristics are indeterminate.

These facts present a conundrum to multi-asset investors considering an allocation to BTC. In this article, we approach the BTC allocation question from the perspective of multi-asset investors who are sufficiently optimistic about the prospects for BTC to consider allocating some portion of their portfolios to BTC. We define multi-asset investors as those who have gone through the portfolio selection exercise and carefully considered the return and risk trade-offs of a variety of risk-efficient portfolios and ultimately selected a portfolio that is aligned with their specific risk preferences. As such, the multi-asset investors we consider temper their optimism about the prospects for BTC against their desire to effectively manage the risk implications that result from BTC allocations.

Given the uncertain nature of BTC's price characteristics and its high volatility, how should multi-asset investors consider the inclusion of BTC in their portfolios? To provide insights into this question, we first explore each of the three points above from the perspective of a multi-asset investor. We then assess the historical performance characteristics of a variety of multi-asset portfolios with BTC allocations and then consider 10,000 alternative "histories" for these portfolios through a block-bootstrap simulation exercise. In assessing allocations, we do not rely on any specialized portfolio optimization methods or the consideration of alternative investor utility functions. Rather, we simply construct stock/bond portfolios with specified allocations to BTC and assess the return and risk implications of those allocations. The incremental benefits and risks of BTC allocations are evaluated relative to an investor's initial portfolio. The purpose of this analysis is to approach the BTC allocation question objectively and to share key insights that can be used by multi-asset investors to inform possible BTC allocations.

## AN OVERVIEW OF BTC RETURNS, RISKS, AND CORRELATIONS

As this article is directed to multi-asset investors considering BTC, it would be useful to first gain an understanding of the characteristics of BTC across the three dimensions most relevant to multi-asset investors: return, risk, and correlations. As previously mentioned, BTC was first launched in January 2009. That means that there is a very limited history with which to inform forward-looking views. It is also the case that BTC was the first cryptocurrency introduced to the world. We make this point to note that both BTC and the systems that have facilitated the exchange (purchase and sale) of the instrument were all nascent and untested. Since 2009, the adoption

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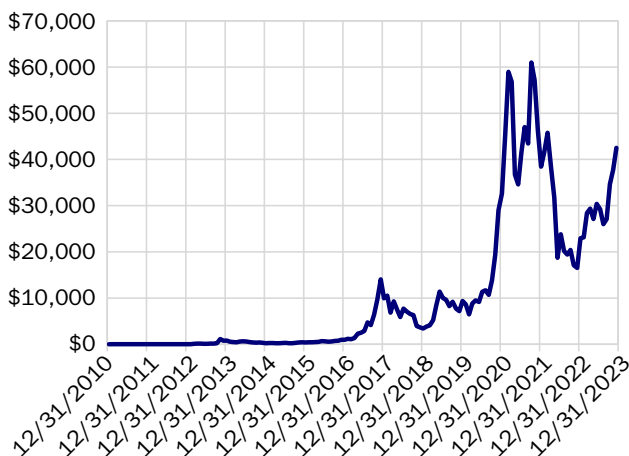
<sup>1</sup>For example, R. Bernstein, "Bitcoin ETFs Are a Siren Song, Not Proof of Concept," *Financial Times*, January 12, 2024, and ARK Invest "Big Ideas 2024" offer sharply contrasting views on the viability of Bitcoin as an investment.

of BTC and the systems that support pricing and marketability have improved notably. As such, understanding the dynamics of the evolution of these aspects of BTC will be useful in forming views for portfolio construction.

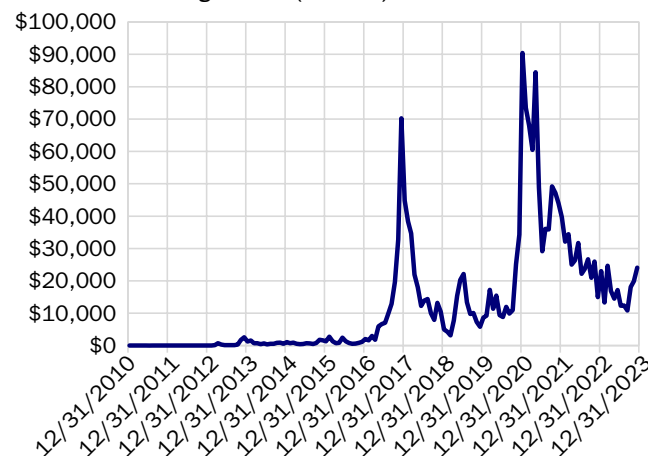
In Exhibit 1, Panels A and B, we present a price history of BTC along with corresponding trading volumes. Unfortunately, the significant jumps in BTC prices seen in Panel A directly affect the trading volumes shown in Panel B. This makes gaining an understanding of the evolution of dynamics across either of these series nearly impossible. To remove the impact of the significant variability in BTC prices, we present the cumulative percentage change of the same two series in logarithmic terms in Panels C and D. This results in time series plots in which the vertical (y-axis) distances represent equal percentage changes. For example, a change in the price of BTC from \$2 to \$4 would look substantially different from a change in price from \$20,000 to \$40,000 in Panel A, even though these changes, in percentage terms, are identical. In Panel C, these two price changes would represent equal vertical distances.

**EXHIBIT 1**  
**BTC Price and Trading Volume**

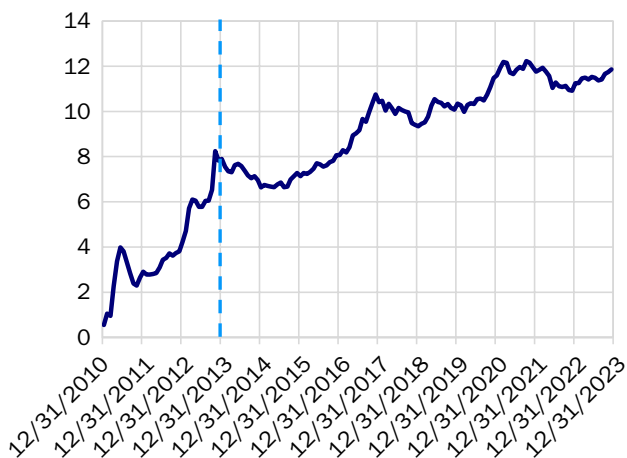
Panel A: BTC Price



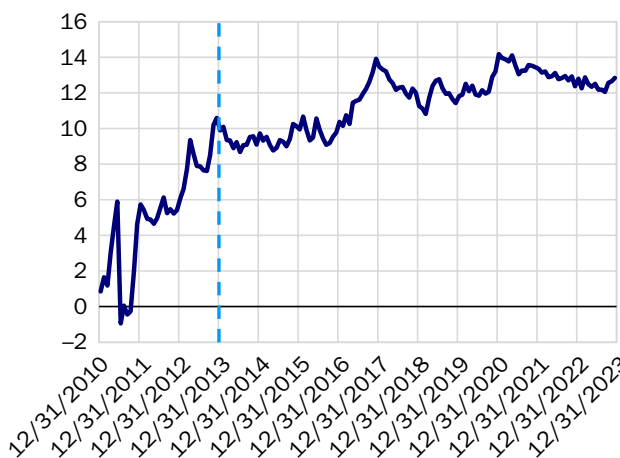
Panel B: BTC Trading Volume (\$million)



Panel C: Log of Cumulative BTC Percent Change in Price



Panel D: Log of Cumulative BTC Percent Change in Trading Volume



**NOTE:** Sample period for BTC price and trading volumes shown is 12/31/2010 through 12/31/2023.

**SOURCE:** Bloomberg, Galaxy, [bitcoinity.org](https://www.bitcoinity.org).

With Panels C and D of Exhibit 1, we can now gain a better understanding of how prices and trading volumes have evolved.

An examination of the log BTC price changes and log trading volume changes shows what appears to be indicative of moderation in price increases and trading volumes. Looking at the period from 12/31/2010, the beginning of values charted, through 12/31/2013, we see a substantial change from 0.55 to 7.82 in log terms, or \$0.30 to \$746.89 in discrete terms. The dramatic increase in the price of BTC during this period also corresponds to a dramatic increase in daily trading volume from 0.85 to 10.60 in log terms, or \$148,000 to \$2,533,670,949 in discrete terms.

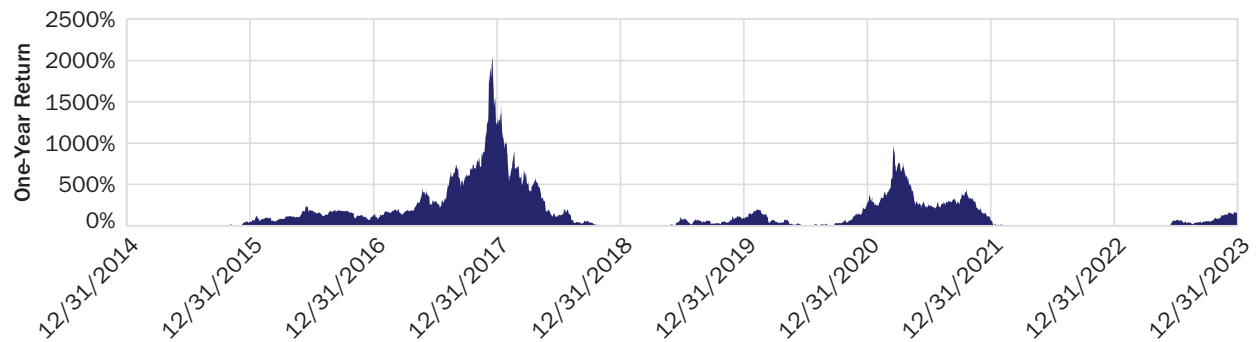
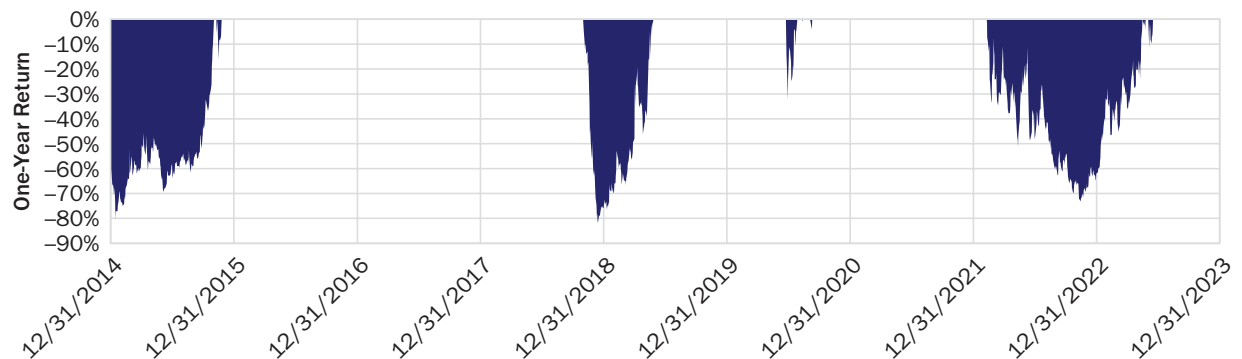
The dramatic increase during this initial period accounts for a substantial portion of the price and trading volume changes for BTC over the entire period analyzed. This initial period for price and volume dynamics is substantially different from what BTC has exhibited since and is not likely to recur. In fact, this is made all the clearer by an examination of the dramatic differences in the distributional properties of BTC returns pre-2014 (skew: 1.2; kurtosis: 11.7) and post-2014 (skew: 0.4; kurtosis: 5.8). These values translate to a pre-2014 period characterized by a fat-tailed distribution with a significant number of positive outliers and a post-2014 period characterized by a fat-tailed distribution (although much less so than pre-2014) with big positive and negative outliers that are approximately symmetric (normally distributed). As such, we believe the post-2014 history of BTC returns is likely to be much more representative of what investors will experience.

Having identified a period of representative BTC returns for consideration as part of our analysis, we can calculate some statistics to summarize the characteristics of BTC during that period. The average annual (arithmetic) return for BTC during this period is 63%, the average annual compound (geometric) return is 48%, and the standard deviation (volatility) is 69%. An assessment of these values leads one to recall the apocryphal example of the statistician that drowned while crossing a river that was, on average, only three feet deep (Savage 2009). Hidden in the estimated average depth of the river was the fact that the river was substantially deeper than three feet in the middle and notably shallower than three feet in the areas closer to the riverbanks. That information would have been useful for the statistician to consider before crossing the river. As such, an analysis of BTC return and risk dynamics during the representative period would also be helpful in establishing a better understanding of what BTC investors are likely to experience.

In Exhibit 2, we present the history of the daily rolling one-year returns for BTC beginning 12/31/2013 and extending through 12/31/2023, broken out by positive one-year returns (Panel A) and negative one-year returns (Panel B). It is useful to focus on positive and negative returns separately as there are significant scaling differences. Negative returns are bound by a maximum loss of 100%, whereas gains are theoretically unlimited and not equivalently bound. Indeed, in the case of BTC, simply showing rolling one-year returns would result in the size of losses being dwarfed by the significant gains BTC has exhibited.

An examination of Exhibit 2, Panel A, provides a clear indication of why BTC has garnered investor interest to the extent it has. Here we see some one-year return periods that were well above 1,000% and a nontrivial number of one-year periods that exceeded 100%. It is likely that investors' interest in BTC would be much more tempered if its returns were more aligned with those of other traditional financial assets. That said, Panel B of Exhibit 2 does offer some balance to the BTC argument and provides reason for some pause in considering BTC for investors' portfolios. Here, we see several one-year periods in which losses have exceeded 70%.

Taleb (2021) puts forth the view that many cryptocurrencies are assets with a fair value of zero and positive probability of ruin. However, it isn't necessary to hold such a negative view of cryptocurrencies to understand that the sizing of a cryptocurrency

**EXHIBIT 2****Rolling One-Year BTC Returns****Panel A: Positive One-Year Returns****Panel B: Negative One-Year Returns**

**SOURCE:** Bloomberg, Galaxy, Invesco. Sample period for BTC returns shown is 12/31/2013 through 12/31/2023.

allocation is a critical part of risk management. Investors considering a BTC allocation would certainly not expect it to go to zero, but they should expect that it will likely exhibit significant losses, as shown in Panel B. The best way to manage the possibility of those losses is to size the portfolio allocation to BTC such that losses, should they occur, fall within the investor's behavioral and financial tolerances and do not result in an irreparable impairment in the ability to achieve important financial objectives. In short, don't allocate more than you are willing to lose. The point here is that risk management is more than volatility management. However, this does not mean that understanding the risk of BTC, in terms of volatility, is not important. It provides an indication of how likely we are to experience an asset's expected (arithmetic mean) return.

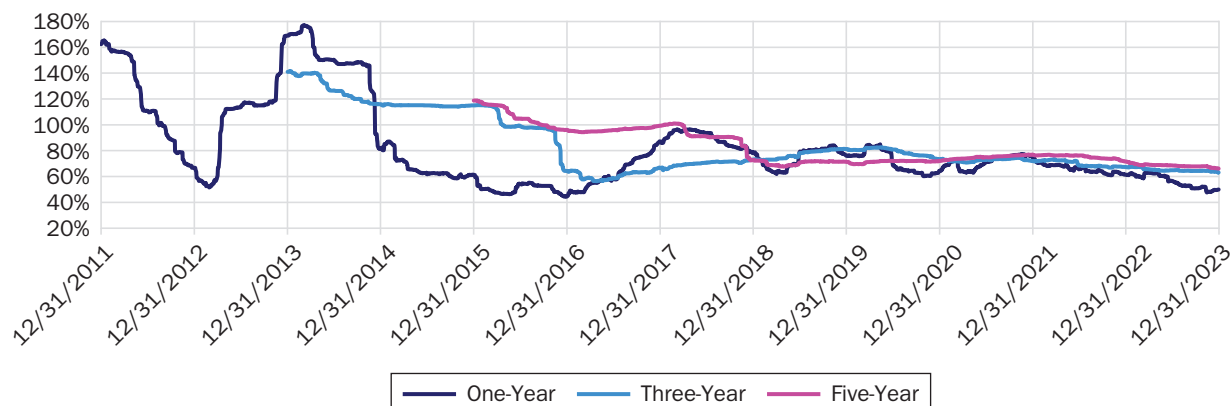
In Exhibit 3, we present the rolling one-, three-, and five-year annualized volatility (standard deviation) of BTC for the period from 12/31/2010 through 12/31/2023. In examining one-year volatility we can clearly see how the period before 12/31/2013 was substantially different from the remaining history. Looking at the one-year volatility value at 12/31/2014, one year after the end of the initial period we identified as exhibiting dramatic price increases, we can see that volatility had dropped to levels much more in line with what is exhibited throughout the rest of the sample period. In fact, once pre-2014 returns are no longer included in the three- and five-year volatility calculations, annualized BTC volatility has remained relatively stable at values between 60% to 80% through the end of 2023.

This brings us to a review of correlations, which are central to understanding the diversification benefits to multi-asset portfolios offered by BTC. In Exhibit 4, we present rolling three-year correlations for BTC and three sets of assets: equities



## EXHIBIT 3

## Rolling Annualized BTC Volatility



**NOTE:** BTC returns used for this exhibit are from the XBTUSD Index for 2011 and the Bloomberg Galaxy Bitcoin Index thereafter.

**SOURCE:** Bloomberg, Galaxy, Invesco. Sample period for BTC returns shown is 12/31/2010 through 12/31/2023.

in Panel A, fixed income in Panel B, and commodities and gold in Panel C. What is most obvious to note across all three sets of assets is that correlations with BTC exhibited easily discernible changes in the early part of 2020, a period that coincides with the beginning of the COVID-19 pandemic.

Panel A of Exhibit 4 shows rolling correlations with small, large, and global equities. The story here is straightforward—BTC correlations with equities increased notably, more so than any of the other assets shown. While there appear to be some small changes in the most recent period shown, the higher correlations have persisted. Here we present only three types of equities but can confirm that the story is largely the same across a much broader set of equity types with no remarkable differences.

In Panel B of Exhibit 4, we show rolling correlations with US Treasuries, Universal bonds, which represent a broadly diversified bond index, and high yield bonds. Here we see the most significant differences in correlations in high yield bonds after the pandemic occurred. That is consistent with what was seen in equities. Interestingly, the correlation with US Treasuries initially decreased. Only in the most recent period, when there was a substantial increase in interest rates, did correlations begin to revert to pre-pandemic levels. We do not see a similar reversion for the high yield credit exposure. The broadly diversified index was initially unchanged, likely due to offsetting exposures in Treasuries and high yield bonds, and then increased in the most recent period.

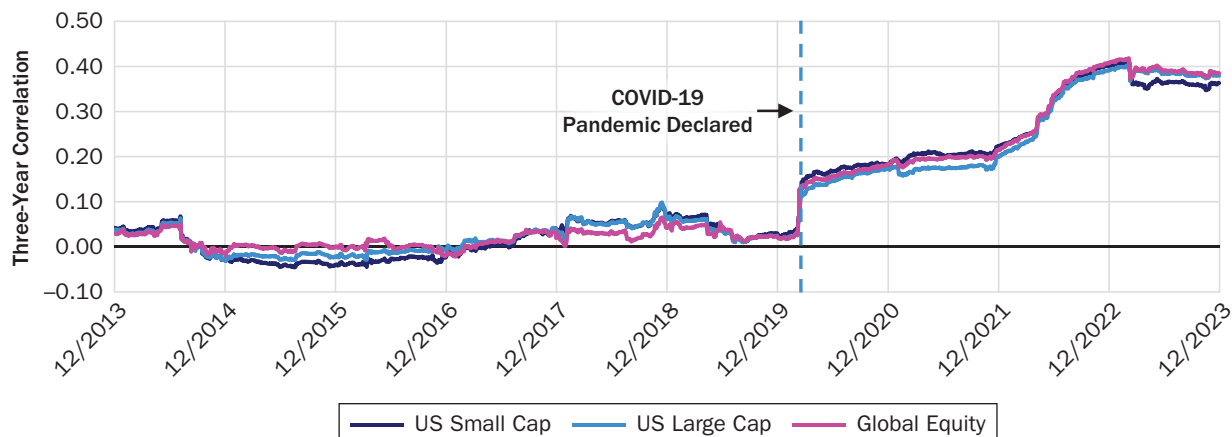
Finally, in Panel C of Exhibit 4, we show rolling correlations to commodities and gold. Here we can see in the early part of the analysis that there were no indications, from a correlation perspective, that BTC was related to either commodities or gold, as some have suggested. In the most recent period, we see, as with other assets, the increase in correlations with the start of the global pandemic. However, there appears to be a trend to reverting to pre-pandemic levels.

The fact that the COVID-19 pandemic had such a notable impact on correlations for all assets considered is problematic to establishing forward-looking views on the relationships between BTC and other assets. What is even more perplexing is that we see a reversion to pre-pandemic levels in assets such as US Treasuries and gold but a persistence in the higher correlation levels in risky assets, such as stocks and high yield bonds, that resulted from the pandemic. Will BTC correlations to risky assets revert to pre-pandemic levels at some point or will they remain at higher levels?

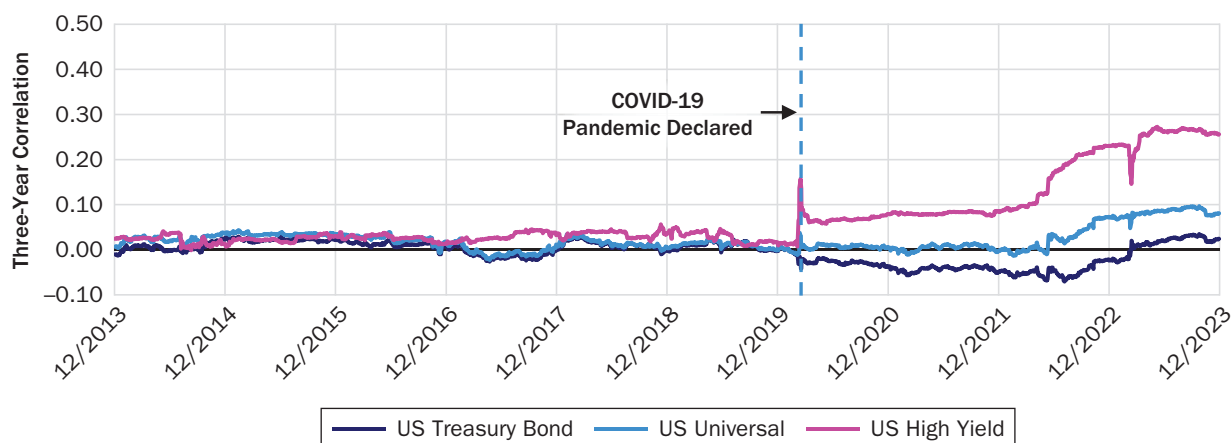


**EXHIBIT 4**  
**Rolling Three-Year Correlations of BTC with Selected Assets**

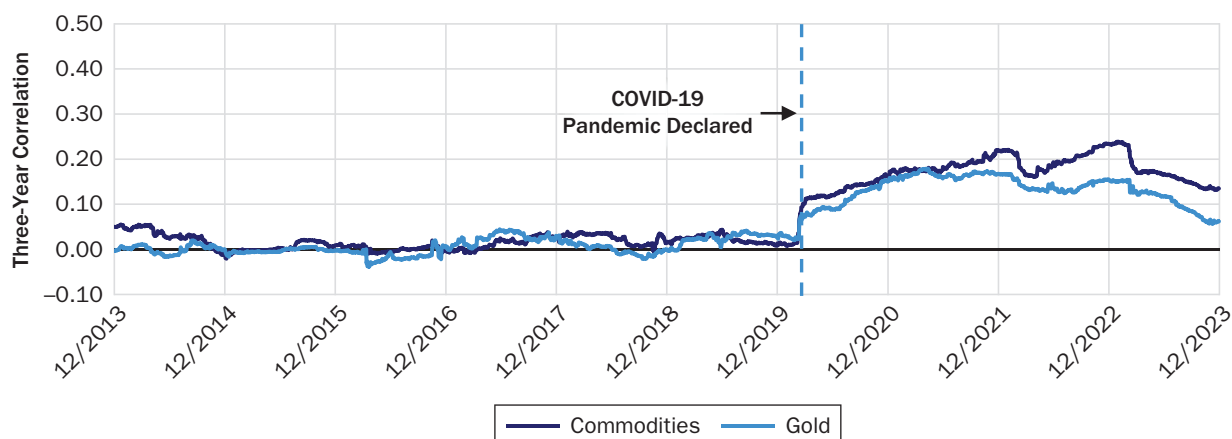
**Panel A: Equities**



**Panel B: Fixed Income**



**Panel C: Gold and Commodities**



**NOTES:** Equities: US Small Cap—Russell 2000; US Large Cap—Russell 1000; Global Equity—MSCI ACWI. Fixed Income: US Treasury Bond—Bloomberg US Treasury; US Universal—Bloomberg US Universal; US High Yield—Bloomberg US Corporate High Yield. Commodities: Bloomberg Commodity Index; Gold: Gold Spot USD. The World Health Organization declared COVID-19 as a pandemic on March 11, 2020.

**SOURCE:** Bloomberg, FTSE Russell, and MSCI.

The answer to that question will have implications for portfolio construction and, ultimately, for multi-asset portfolio investing outcomes.

An important point about inferring BTC's relationship with other assets during the limited sample period available is that the period includes a global pandemic, massive government fiscal and policy intervention globally, the greatest increase in interest rates in more than 20 years, the most significant increase in inflation in more than 40 years, and instances of stock–bond correlations that were among both the lowest and highest levels historically. Couple that with the fact that BTC has been maturing from an emerging instrument to becoming more mainstream during the period. It should be clear that making reliable determinations about future relationships between BTC and other assets is, at best, a complicated matter. Even analyses of factor exposures or other covariance modeling techniques are likely to produce mathematically correct but practically unreliable inputs for asset allocation purposes.

Having now examined the three dimensions of greatest interest for multi-asset investors, we provide a summary of our findings:

- After an initial period of significant increases (12/31/2010–12/31/2013), BTC prices and trading volumes appear to have moderated.
- BTC has exhibited outsized gains and significant losses that investors must consider carefully before investing.
- BTC annualized volatility appears to have stabilized at somewhere between 60% and 80%, which is high relative to traditional assets but significantly lower than what BTC exhibited before 2014.
- BTC correlations are meaningfully different during the periods before and after the beginning of the COVID-19 pandemic. Some assets appear to be reverting to pre-pandemic levels while others, risk assets in particular, persist at higher levels. Forward-looking estimates based on those data are likely to be unreliable.

## **ASSESSING BTC ALLOCATIONS: APPROACH, METHODOLOGY, AND DATA**

The previous section focused on providing an overview, albeit brief, of relevant characteristics of BTC returns in the context of portfolio analysis. It provides an important backdrop for considering how multi-asset investors might approach the BTC allocation question. For that purpose, there is a considerable and growing body of research focused on addressing the challenges presented by BTC and other cryptocurrencies.

Harvey et al. (2022) provide a general overview of the cryptocurrency space, discuss the significant challenges in valuing cryptocurrencies, provide perspective on the relevant characteristics of crypto returns (volatility and correlations), and present a variety of practical considerations for investors. Ang, Morris, and Savi (2023) focus on the skewness aspects of BTC returns and consider various investor utility preferences to identify optimal allocations of 3% or less, given specific investor beliefs. Gaskin et al. (2023) provide a review of the challenges with considering cryptocurrencies in portfolios. They focus on various methods of estimating relevant return distribution characteristics, consider methods for reducing the estimation error in optimization parameters, and then assess various optimization approaches to determine which might be best for the case in which cryptocurrencies are considered for inclusion in portfolios. Czaronis et al. (2022) consider issues with correlation estimates that fail to capture differences in upside and downside correlations as well as the magnitude in directional returns and finally consider different utility preferences in determining

minimum return thresholds required for BTC allocations depending on the investor utility function used. Finally, Hubrich (2022) provides an overview of BTC portfolio allocation research to date, concluding that optimization estimates from the available BTC return history result in entirely unrealistic allocations. He then focuses on determining the BTC return level required to rationalize a 1% or 5% allocation to BTC. The findings show that small allocations to BTC can be justified even if one believes BTC is likely to trend toward zero. However, Hubrich also finds that these and other results are critically affected by rebalancing frequency.

There are many interesting and useful insights to be extracted from the existing literature on portfolio allocation to cryptocurrencies, generally, and to BTC, specifically. However, the practical application of many of the methods considered is limited. Implementing complex optimization techniques or incorporating alternative utility functions simply to include a small allocation to BTC or other cryptocurrencies is beyond the quantitative capabilities of many investors and the organizational/policy limitations of others, and are unlikely to be adopted. More pragmatic guidance on BTC allocation is required.

While Markowitz (2012) and Markowitz and Blay (2012) provide clarification on the “great confusion,” or rather, the confusion between the necessary and sufficient conditions for the use of mean–variance analysis in practice, it is not necessary to actually conduct a portfolio analysis to realize that BTC will play a prominent role in any portfolios that are produced without imposing constraints. Historical BTC returns are several times that of even the highest returning traditional assets and exhibit much more favorable return-to-risk (RTR) characteristics. Add historical correlations that are near zero, less than 0.50, or merely unreliable, and it is clear BTC would be problematic for any standard optimization techniques. Even more sophisticated techniques must contend with unreliable optimization parameters. As it stands today, a pragmatic approach to considering BTC allocations should avoid optimization altogether. There simply isn’t enough history to make long-term determinations about BTC return characteristics. As Markowitz himself stated, “in practice, formal analysis must frequently resort to approximation, and call upon judgement.”

So, how are multi-asset investors to approach considering BTC allocations? A reasonable and common starting point is to examine the composition of the market. Assuming investors and markets offer useful information about efficient capital allocation, understanding what proportion of total capital is allocated to an asset can serve as a useful starting point to inform allocation decisions. Sharpe (2010) explains that an allocation to assets in the same amount as they are held in the market portfolio is *macro-consistent*, that is, it is the allocation that all investors can hold. In Exhibit 5 we provide a rough estimate of the market capitalization of major assets that make up a practical representation of the global investable market portfolio. Here we find that an allocation of 1% to crypto assets is aligned with the notion of a macro-consistent allocation. In fact, the market capitalization of crypto assets is in line with assets held in private markets that are regularly considered for inclusion in multi-asset portfolios. Of course, Exhibit 5 is an incomplete representation of the theoretical market portfolio that includes ALL market assets. However, it is still suggestive of the idea that investors might consider allocating some portion of their portfolio to BTC versus allocating nothing at all.

To further inform allocations to BTC in multi-asset portfolios, we will first go through the exercise of considering the historical record of a broad range of multi-asset portfolios consisting of stocks, bonds, and BTC during the representative period identified previously (1/1/2014–12/31/2023). We later expand this analysis to consider alternative return outcomes. The assets and representative proxy indexes used for this analysis are as follows. Stocks are represented by the MSCI All Country World

## EXHIBIT 5

## BTC and the Market Portfolio

Asset Class	Market Capitalization (\$trillion)	Percent of Total Market
<b>Fixed Income</b>	<b>128.0</b>	<b>47.8</b>
US	51.1	
Developed Markets ex-US	48.7	
Emerging Markets	28.2	
<b>Equity</b>	<b>106.0</b>	<b>39.6</b>
US	45.4	
Developed Markets ex-US	35.6	
Emerging Markets	25.0	
<b>Private Markets</b>	<b>13.2</b>	<b>4.9</b>
Private Equity/Venture Capital	8.3	
Private Debt	1.7	
Real Estate	1.7	
Infrastructure and Natural Resources	1.5	
<b>Commodities</b>	<b>17.9</b>	<b>6.7</b>
Gold	16.4	
Silver	1.5	
<b>Crypto</b>	<b>2.7</b>	<b>1.0</b>
Bitcoin	1.4	
Ether and Others	1.3	
<b>TOTAL</b>	<b>267.8</b>	<b>100.0</b>

**SOURCE:** Market size data are collated from the following sources: fixed income and equity—SIFMA 2024 Capital Market Outlook, private markets—McKinsey Global Private Markets Review 2024, gold and silver—The World Gold Council and [companiesmarketcap.com](https://companiesmarketcap.com), crypto—[Coinmarketcap.com](https://Coinmarketcap.com).

Equity Index, bonds are represented by the Bloomberg Barclays US Universal Bond Index, and BTC is represented by the Bloomberg Galaxy Bitcoin Index.

We consider six initial stock/bond allocations ranging from 100% stocks to 100% bonds, with incremental allocations between stocks and bonds changing in 20% increments. For each initial stock/bond allocation we assess allocations to BTC ranging from 1% to 10% in 1% increments by proportionately shrinking the initial allocation to allow for the funding of the BTC allocation. Funding BTC in this manner is aligned with our empirical analysis and the fact that reliable covariance estimates required for determining optimal funding sources are not currently available. In total, we evaluate 66 unique portfolios. Analyzed portfolios are rebalanced annually. We will provide more detail on the importance of rebalancing frequency later.

Given that lower returns to BTC is a very plausible outcome, we assess BTC allocations using two different scenarios. The first uses historical BTC returns. The second assumes that BTC returns were 50% of what they were historically. This is done by adjusting BTC returns so that the arithmetic average return over the analysis period is equal to 50% of the historical arithmetic average. This does not affect the distributional properties of the return history, only its point of central tendency.

To assess the benefits of various BTC allocations we relate the familiar portfolio Return-to-Risk (RTR) metric to an investor's initial portfolio risk preferences by balancing incremental benefits from BTC allocations, in terms of improvements in RTR, against the incremental risks added as a function of the BTC allocations. This is done by translating the incremental benefits to multiples of the initial portfolio's RTR ratio. For example, an initial portfolio might have an RTR ratio (calculated as return/risk) of 0.62.

Adding a 1% allocation to BTC increases RTR to 0.77. Dividing 0.77 by 0.62, we arrive at a multiple of 1.24. This means that the allocation to BTC increased the initial portfolio's RTR by 24%. Incremental risks are also translated to multiples of the initial portfolio's risk. The RTR multiple is then divided by the risk multiple to arrive at a benefit-to-risk (BTR) metric for each portfolio. A complete description of the BTR metric is provided in the appendix.

Conceptually, the BTR metric will be greater than one if benefits accrued from additional BTC allocations are greater than the incremental risks added. The BTR metric will have achieved a maximum at the point before the incremental risk from a BTC allocation is greater than the incremental benefit. We will highlight the maxima for each of the six sets of portfolio allocations. We view these maxima as suggestive of a rational maximum BTC allocation. That is not to say that additional benefits from increasing BTC allocations are not possible, only that increased risk is not commensurate with the increased benefit. We also note that identified maxima are not to be considered suggested allocations. With each increment in BTC allocation comes an increase in risk that an investor should carefully consider. In terms of risk, it would be fair to say that a little BTC goes a long way. We should also recall the importance of BTC allocation sizing as a critical part of risk management. Investors should not allocate more than they are willing to lose.

A key element of our assessment of BTC allocation decisions is an investor's starting risk preferences. The rationale for that approach is as follows: If an investor had been willing to accept a higher level of portfolio risk initially, they would have already invested in a higher-risk portfolio. Put another way, investors who seek to maximize return for some given level of risk would have already selected a portfolio that aligned with their risk preferences. While we don't argue that there may be some non-financial utility benefits related to BTC allocations, here we assume that an investor requires some financial benefit for allocating to BTC. In other words, investors don't seek to allocate to BTC just for the sake of having a BTC allocation, they must have a financial incentive to increase risk from their initial allocation.

An important point to note about considering the BTR implications of allocations to BTC from a specific starting portfolio allocation is that this assumes specific investor initial risk preferences and explores outcomes from extending those preferences to riskier portfolios. Investors considering accepting the additional risk resulting from BTC allocations would be well served to also explore the alternative of investing in a higher risk allocation without BTC. It could be that the allocation to BTC is warranted given the investor's return expectations. It could also be that the investor might be better served by simply allocating to a higher risk portfolio of traditional capital market assets. Assessing this trade-off is out of the scope of this article given that such a comparison of the two alternative portfolio outcomes relies largely, if not exclusively, on an investor's unique forward-looking return, risk, and correlation assumptions for capital market assets and BTC. However, this trade-off should be evaluated in practice.

## **BTC ALLOCATIONS: THE HISTORICAL PERSPECTIVE**

Exhibit 6 provides summary statistics for all 66 portfolios considered using the historical return of BTC. The 0% BTC allocation column is highlighted in gray to provide easy reference to the initial portfolio's summary statistics. The risk multiples, RTR multiples, and BTR metrics for this initial portfolio are all 1. Values for each of the metrics scale up from this starting point. BTR maxima are highlighted for each set of portfolios in light blue. As previously noted, these maxima are not suggested allocations, they merely represent the point at which additional BTC allocations

**EXHIBIT 6**  
**Historical Multi-Asset Portfolios Outcomes**

		Portfolio Summary Statistics											
		BTC Allocation (%)											
		0	1	2	3	4	5	6	7	8	9	10	
Initial Portfolio Stock/Bond Allocation	100/0	Return (%)	7.41	8.80	10.10	11.35	12.54	13.68	14.77	15.83	16.85	17.83	18.79
		Risk (%)	14.48	14.57	14.83	15.20	15.64	16.13	16.66	17.21	17.77	18.35	18.93
		Return-to-Risk (RTR)	0.51	0.60	0.68	0.75	0.80	0.85	0.89	0.92	0.95	0.97	0.99
		Risk Multiple	1.00	1.01	1.02	1.05	1.08	1.11	1.15	1.19	1.23	1.27	1.31
		RTR Multiple	1.00	1.18	1.33	1.46	1.57	1.66	1.73	1.80	1.85	1.90	1.94
		Benefit-to-Risk (BTR)	1.00	1.17	1.30	1.39	1.45	1.49	1.51	1.51	1.51	1.50	1.48
	80/20	Return (%)	6.42	7.84	9.19	10.47	11.69	12.85	13.98	15.05	16.10	17.10	18.08
		Risk (%)	11.47	11.63	12.00	12.50	13.08	13.72	14.37	15.05	15.73	16.41	17.09
		Return-to-Risk (RTR)	0.56	0.67	0.77	0.84	0.89	0.94	0.97	1.00	1.02	1.04	1.06
		Risk Multiple	1.00	1.01	1.05	1.09	1.14	1.20	1.25	1.31	1.37	1.43	1.49
		RTR Multiple	1.00	1.21	1.37	1.50	1.60	1.67	1.74	1.79	1.83	1.86	1.89
		Benefit-to-Risk (BTR)	1.00	1.19	1.31	1.37	1.40	1.40	1.39	1.36	1.33	1.30	1.27
	60/40	Return (%)	5.40	6.87	8.25	9.56	10.81	12.01	13.16	14.26	15.33	16.35	17.35
		Risk (%)	8.68	8.92	9.44	10.12	10.87	11.66	12.46	13.26	14.06	14.84	15.62
		Return-to-Risk (RTR)	0.62	0.77	0.87	0.95	0.99	1.03	1.06	1.08	1.09	1.10	1.11
		Risk Multiple	1.00	1.03	1.09	1.17	1.25	1.34	1.44	1.53	1.62	1.71	1.80
		RTR Multiple	1.00	1.24	1.40	1.52	1.60	1.66	1.70	1.73	1.75	1.77	1.79
		Benefit-to-Risk (BTR)	1.00	1.20	1.29	1.30	1.28	1.23	1.18	1.13	1.08	1.04	0.99
	40/60	Return (%)	4.35	5.86	7.29	8.63	9.92	11.14	12.32	13.45	14.54	15.59	16.60
		Risk (%)	6.21	6.57	7.29	8.18	9.12	10.08	11.01	11.93	12.83	13.70	14.55
		Return-to-Risk (RTR)	0.70	0.89	1.00	1.06	1.09	1.11	1.12	1.13	1.13	1.14	1.14
		Risk Multiple	1.00	1.06	1.17	1.32	1.47	1.62	1.77	1.92	2.07	2.21	2.34
		RTR Multiple	1.00	1.28	1.43	1.51	1.55	1.58	1.60	1.61	1.62	1.63	1.63
		Benefit-to-Risk (BTR)	1.00	1.21	1.22	1.15	1.06	0.97	0.90	0.84	0.78	0.74	0.70
20/80	Return (%)	3.26	4.83	6.30	7.68	9.00	10.26	11.46	12.61	13.73	14.80	15.84	
	Risk (%)	4.44	4.92	5.86	6.95	8.05	9.13	10.17	11.16	12.12	13.05	13.94	
	Return-to-Risk (RTR)	0.73	0.98	1.07	1.11	1.12	1.12	1.13	1.13	1.13	1.13	1.14	
	Risk Multiple	1.00	1.11	1.32	1.56	1.81	2.05	2.29	2.51	2.73	2.94	3.14	
	RTR Multiple	1.00	1.34	1.46	1.50	1.52	1.53	1.53	1.54	1.54	1.54	1.55	
	Benefit-to-Risk (BTR)	1.00	1.21	1.11	0.96	0.84	0.74	0.67	0.61	0.57	0.53	0.49	
0/100	Return (%)	2.15	3.77	5.28	6.71	8.06	9.35	10.58	11.76	12.90	14.00	15.06	
	Risk (%)	4.19	4.64	5.62	6.74	7.87	8.96	10.01	11.02	11.98	12.91	13.81	
	Return-to-Risk (RTR)	0.51	0.81	0.94	0.99	1.02	1.04	1.06	1.07	1.08	1.08	1.09	
	Risk Multiple	1.00	1.11	1.34	1.61	1.88	2.14	2.39	2.63	2.86	3.08	3.30	
	RTR Multiple	1.00	1.58	1.83	1.94	2.00	2.03	2.06	2.08	2.10	2.11	2.13	
	Benefit-to-Risk (BTR)	1.00	1.43	1.36	1.20	1.06	0.95	0.86	0.79	0.73	0.69	0.64	

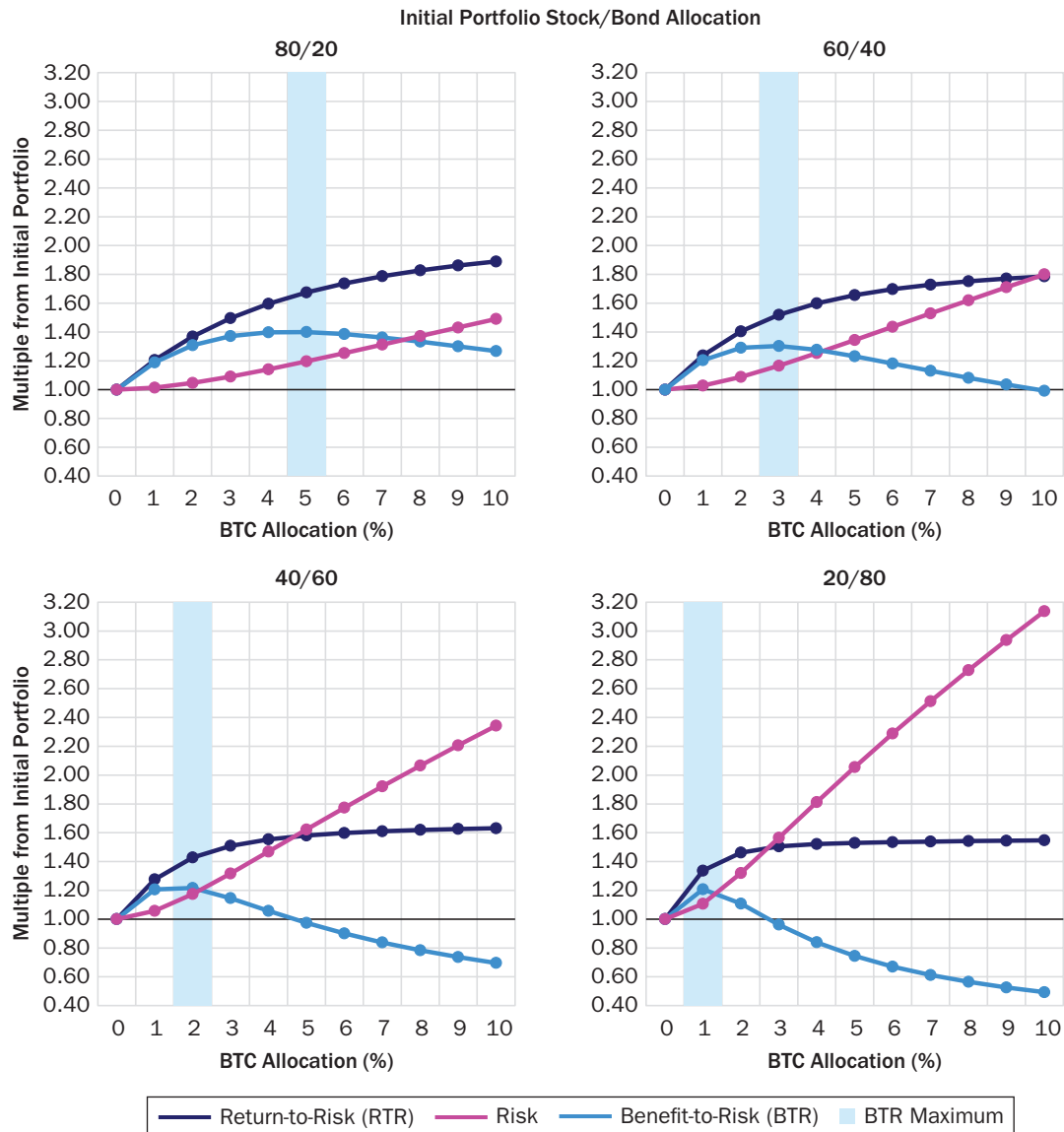
SOURCE: Bloomberg, Galaxy, MSCI, Invesco. Based on historical returns beginning 1/1/2014 through 12/31/2023.

begin to add more risk than benefit. For example, in looking at the 80/20 set of portfolios we see that the maximum is reached at a BTC allocation of 5%. While the RTR multiple of 1.67 indicates a notable 67% increase in RTR, the risk multiple of 1.20 indicates that the benefit comes with a 20% increase in risk relative to the initial 80/20 portfolio.

Exhibit 7 presents the BTR analysis in graphical form for the multi-asset subset of portfolios considered. The top two graphs in the exhibit are the more aggressive, higher equity allocations. The bottom two graphs are the more conservative, higher bond allocations. As done previously, BTR maxima are highlighted in light blue.



**EXHIBIT 7**  
**Historical Multi-Asset Portfolios Outcomes**



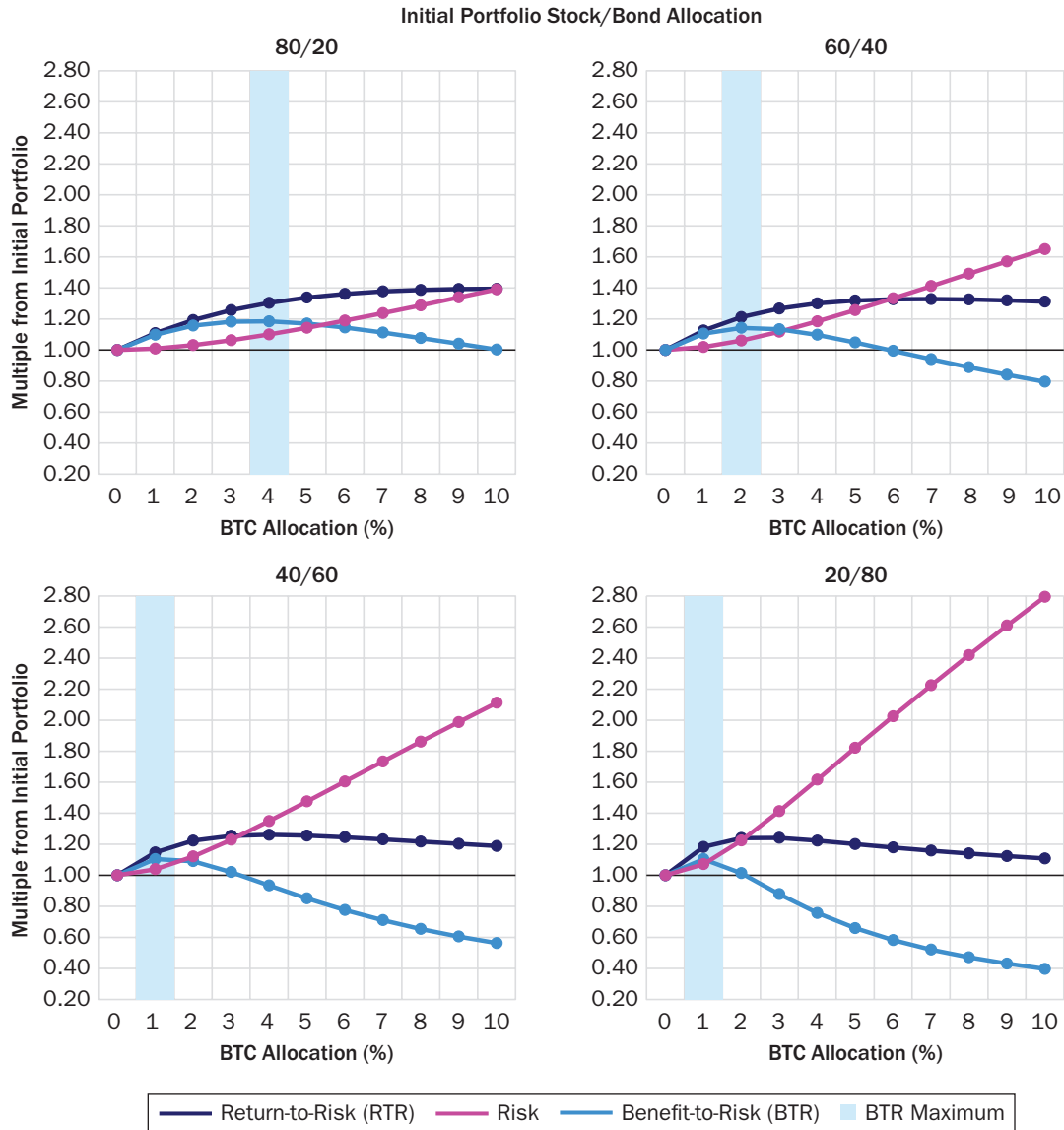
**SOURCE:** Bloomberg, Galaxy, MSCI, Invesco. Based on historical returns beginning 1/1/2014 through 12/31/2023.

These figures allow us to see the interplay more clearly between improvements in RTR and increases in risk. In looking at the conservative 20/80 allocation, we see that the maximum is reached at a 1% BTC allocation. We also see that increasing the BTC allocation to 2% would continue to improve RTR from the initial portfolio's RTR ratio. However, the incremental increase in risk is substantially greater than the incremental increase in RTR. Referencing Exhibit 6 we see that moving to a 2% BTC allocation means accepting a 32% increase in risk.

Exhibit 8 presents the graphical BTR analysis for the scenario in which BTC returned only 50% of its historical return (a tabular presentation of the results is included in Appendix Exhibit A1). As expected, we see the RTR multiple curves flatten out relative to the first scenario. We also see that maxima occur at lower BTC allocations for all portfolio sets except for the most conservative 20/80 case, which

**EXHIBIT 8**

**Historical Multi-Asset Portfolios Outcomes—Assuming 50% of Historical BTC Return**



SOURCE: Bloomberg, Galaxy, MSCI, Invesco. Based on historical returns beginning 1/1/2014 through 12/31/2023.

had previously also occurred at 1%. What is notable with these results is that even with a substantial decrease in BTC returns, we still see incremental benefits greater than the incremental risks for holding BTC.

**BTC AND REBALANCING**

The historical results presented are based on portfolios that are rebalanced annually. Given the impact on portfolio risk that can result from even small allocations to BTC, it is critically important to rebalance portfolio allocations regularly. The potential for significant run-ups in BTC means that allocations can quickly extend well beyond intended allocations and expose investors to much more significant

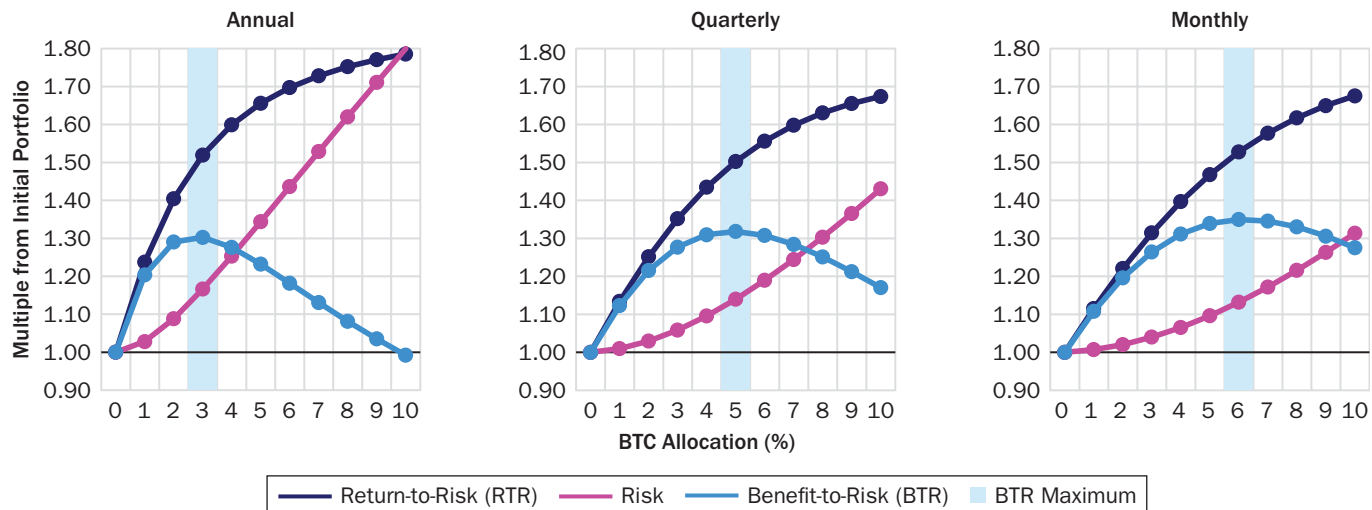
drawdowns than desired. Hougan and Lawant (2021) provide an overview of the importance of rebalancing portfolios with BTC allocations, and Hubrich (2022) also points out the significance of the rebalancing frequency decision on estimates of return and risk for portfolios holding BTC. To provide some insight on just how significant a role the rebalancing decision can have on the question of BTC allocations, we present an analysis of rebalancing frequency on portfolio outcomes.

In Exhibit 9 we present a rebalancing analysis on an initial 60/40 portfolio using annual, quarterly, and monthly rebalancing frequencies. In Panel A, we present the

**EXHIBIT 9**

**Historical 60/40 Portfolio Outcomes Using Different Rebalancing Frequencies**

**Panel A: Assessment of Benefit-to-Risk (BTR)**



**Panel B: Portfolio Summary Statistics**

		BTC Allocation (%)										
		0	1	2	3	4	5	6	7	8	9	10
Annual	Return (%)	5.40	6.87	8.25	9.56	10.81	12.01	13.16	14.26	15.33	16.35	17.35
	Risk (%)	8.68	8.92	9.44	10.12	10.87	11.66	12.46	13.26	14.06	14.84	15.62
	Return-to-Risk (RTR)	0.62	0.77	0.87	0.95	0.99	1.03	1.06	1.08	1.09	1.10	1.11
	Risk Multiple	1.00	1.03	1.09	1.17	1.25	1.34	1.44	1.53	1.62	1.71	1.80
	RTR Multiple	1.00	1.24	1.40	1.52	1.60	1.66	1.70	1.73	1.75	1.77	1.79
	Benefit-to-Risk	1.00	1.20	1.29	1.30	1.28	1.23	1.18	1.13	1.08	1.04	0.99
Quarterly	Return (%)	5.48	6.27	7.06	7.84	8.61	9.38	10.14	10.90	11.64	12.39	13.12
	Risk (%)	8.70	8.79	8.96	9.21	9.53	9.92	10.35	10.83	11.34	11.88	12.45
	Return-to-Risk (RTR)	0.63	0.71	0.79	0.85	0.90	0.95	0.98	1.01	1.03	1.04	1.05
	Risk Multiple	1.00	1.01	1.03	1.06	1.10	1.14	1.19	1.24	1.30	1.37	1.43
	RTR Multiple	1.00	1.13	1.25	1.35	1.44	1.50	1.56	1.60	1.63	1.66	1.67
	Benefit-to-Risk	1.00	1.12	1.22	1.28	1.31	1.32	1.31	1.28	1.25	1.21	1.17
Monthly	Return (%)	5.57	6.26	6.94	7.62	8.30	8.97	9.64	10.30	10.96	11.62	12.27
	Risk (%)	8.80	8.86	8.98	9.15	9.38	9.65	9.96	10.31	10.70	11.12	11.56
	Return-to-Risk (RTR)	0.63	0.71	0.77	0.83	0.88	0.93	0.97	1.00	1.02	1.04	1.06
	Risk Multiple	1.00	1.01	1.02	1.04	1.07	1.10	1.13	1.17	1.22	1.26	1.31
	RTR Multiple	1.00	1.12	1.22	1.31	1.40	1.47	1.53	1.58	1.62	1.65	1.68
	Benefit-to-Risk	1.00	1.11	1.20	1.26	1.31	1.34	1.35	1.35	1.33	1.31	1.28

SOURCE: Bloomberg, Galaxy, MSCI, Invesco. Based on historical returns beginning 1/1/2014 through 12/31/2023.

graphical BTR analysis to facilitate an understanding of the impact on RTR, risk, and, consequently, BTR maxima. Panel B presents a tabular view of the same analysis.

In Panel A, we can clearly see the drastic decrease in the slope of the risk multiple curves as rebalancing frequency increases. We can also see how BTR maxima are affected. Increasing rebalancing allows for consideration of higher allocations to BTC at lower risk multiples. Panel B provides the numerical values to confirm the results. Here we see that by simply increasing the rebalancing frequency from annual to quarterly, portfolio risk is reduced even though the allocation to BTC is higher. In fact, when we run a similar analysis using BTC returns adjusted such that the compound return over the analysis period is 0%, a scenario that assumes that an un-rebalanced BTC allocation would be unchanged at the end of the period, we find that there are still notable benefits to portfolio returns and risk from holding BTC and rebalancing regularly.<sup>2</sup> In any case, transaction costs and tax implications that result from rebalancing should also be considered in determining the implemented rebalancing frequency.

These results are not a specific characteristic of BTC, rather they are a function of what is known as rebalancing benefit or volatility harvesting. Bouchev et al. (2012) demonstrate how portfolio rebalancing can be beneficial to portfolios. Bouchev, Nemtchinov, and Wong (2015) expand on their previous analysis and conclude that all that is required to benefit from rebalancing is multiple assets with differing returns. Chambers and Zdanowicz (2014) provide a detailed analysis of the drivers of the benefits from rebalancing and point to mean reversion as a key driver. Cuthbertson et al. (2016) provide an analysis of the misconceptions around the benefits of rebalancing and guide against the notion of volatility pumping, which is a strategy that deliberately chooses high-volatility assets. The question of how and why benefits from rebalancing exist is complicated. However, that benefits from rebalancing do exist is not in doubt. We can now expand our analysis to move beyond relying on the single historical sample of BTC returns we have.

## **BTC ALLOCATIONS: CONSIDERING ALTERNATIVE “HISTORIES”**

In his 1959 book on portfolio selection, Harry Markowitz explained that “when past performances of securities are used as inputs, the outputs of the analysis are portfolios which performed particularly well in the past” (Markowitz 1959). Unfortunately, much of the guidance on including BTC in portfolios is based on 10 years or less of trailing historical returns, largely because BTC has a very limited usable return history. An understanding of this history, as we have explored, is important and useful to informing forward-looking expectations. However, investors should also recognize the limitations of summarizing the characteristics of BTC through return, risk, and correlation estimates calculated from the single limited historical sample.

If we are to more fully conceptualize the risk implications of investing in BTC, it would be wise to consider a more holistic understanding of what risk means. One of the most concise explanations of risk was put forth by Elroy Dimson and Paul Marsh, who explained that “risk means that more things can happen than will happen” (Dimson and Marsh 1982). This notion extends the concept of risk well beyond the idea of volatility. So, how might investors consider outcomes that could happen beyond the single historical instance that did happen? For that, investors

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<sup>2</sup>A BTC 0% compound (geometric) annual return corresponds to a 24.3% annual (arithmetic) return. Transaction costs and taxes that result from rebalancing are not considered in our analysis.

often turn to simulation methods. Simulated returns allow for the consideration of investing outcomes that could happen based on the characteristics of assets being contemplated. The problem with applying simulation methods is that many require the same parameter estimates used for portfolio construction—returns, standard deviations, and correlations. As previously noted, determining these estimates for BTC is problematic.

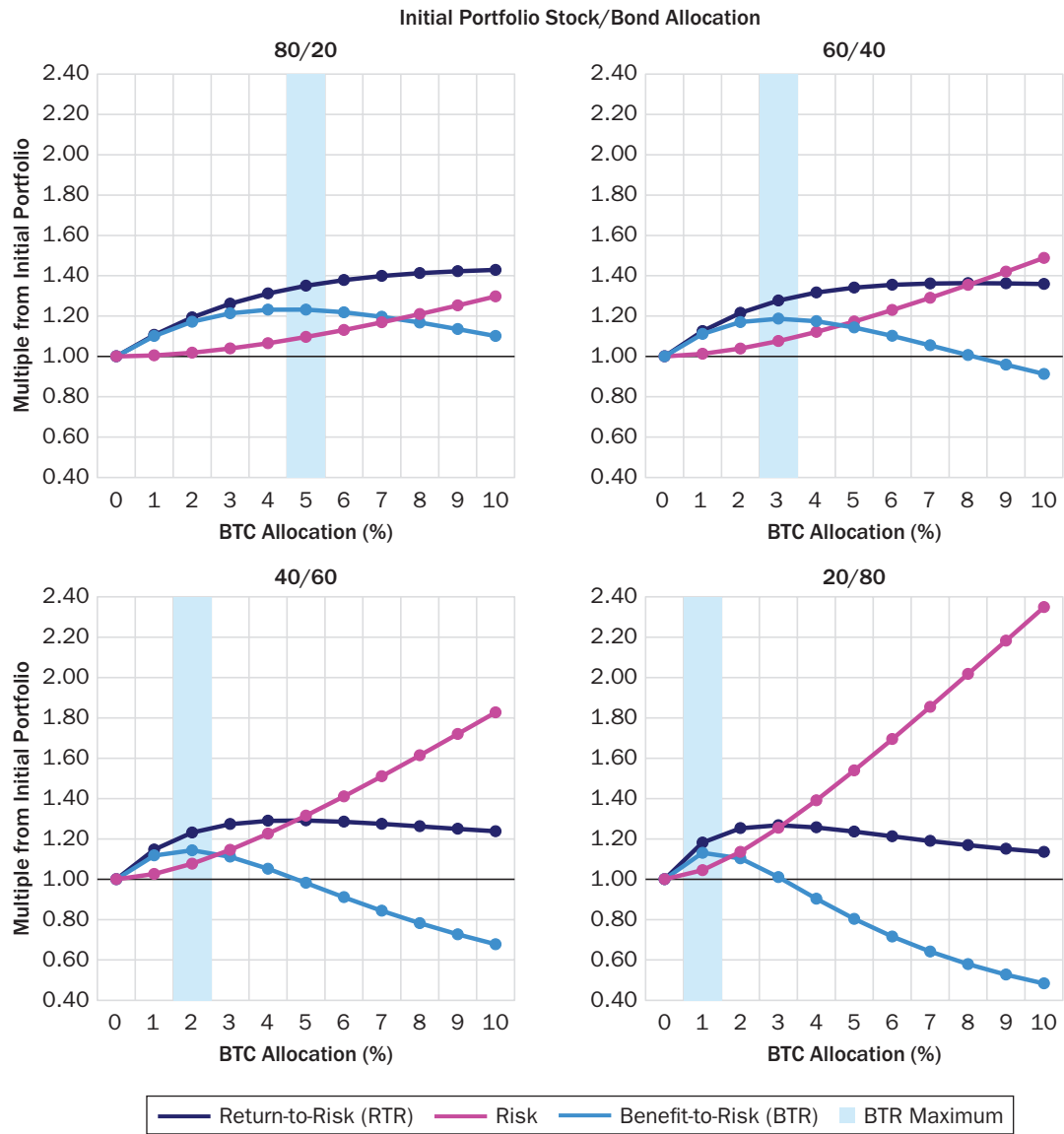
To avoid the limitations of parameter-based simulation methods, we employ block bootstrapping, a commonly used simulation technique that does not require explicit parameter estimates or complicated models of covariance. Instead, the approach randomly samples and combines a series of blocks of returns from the historical sample to create alternative histories that respect the distributional properties and asset relationships that exist within the historical sample. The use of return blocks also allows us to incorporate autocorrelation characteristics that might be lost by drawing single independent periods or employing a model-based approach to simulation.

For our analysis, we randomly draw and combine 10-day blocks from the representative historical sample of stock, bond, and BTC returns (1/1/2014 through 12/31/2023) to create 10,000 alternative sets of one-year histories. We then calculate the return and risk outcomes for the same broad set of multi-asset (stock/bond/BTC) portfolios considered in our historical portfolio analysis. Finally, we apply the BTR framework for assessing BTC allocations using average RTR and risk values calculated from our 10,000 simulations. This exercise allows us to assess the implications of BTC allocations to multi-asset portfolios across a wide range of alternative return histories. In other words, it allows us to consider outcomes that could have happened rather than only the single outcome that did. As we did in our historical assessment of multi-asset portfolios, we also consider the case in which BTC returns are half of what they were in the past.

In Exhibit 10 we present the results of our simulation exercise for the historical BTC return case. Here we find more tempered RTR curves with lower benefits accruing to higher allocations. However, we also find that BTR maxima are the same as what we saw in our historical analysis. Exhibit 11 presents results for the 50% of historical BTC return case. Here we also see tempered RTR curves with some BTC allocations resulting in RTR multiples that fall below initial values. With regard to maxima, results are like the corresponding analysis based on reduced historical returns except for the 80/20 case in which the maximum occurred at a lower BTC allocation (tabular results for both cases are provided in Appendix Exhibits A2 and A3).

To gain a better understanding of the implications of BTC allocations across alternative return paths, we can compare the simulated BTR outcomes with the corresponding historical outcomes. In Exhibit 12 we present a BTR comparison in terms of the ratio of simulated BTR to historical BTR. For this analysis we consider the case that assumes historical BTC returns. Here we find that, on average, simulated outcomes for more conservative, bond-heavy, portfolios exhibited more favorable results than more aggressive, stock-heavy, portfolios. These results are driven by diversification and rebalancing dynamics that are a function of the low correlations of investment-grade bonds with BTC across the sample period considered. The importance of even a small amount of diversification in extracting the benefits of BTC allocations can be seen in comparing the 80/20 stock/bond set of allocations with those of the 100/0, all equity, set that exhibits substantially worse outcomes. Those results suggest that diversification and rebalancing are critical to extracting value from BTC allocations while also managing risk.

**EXHIBIT 10**  
**Simulated Multi-Asset Portfolios Outcomes**

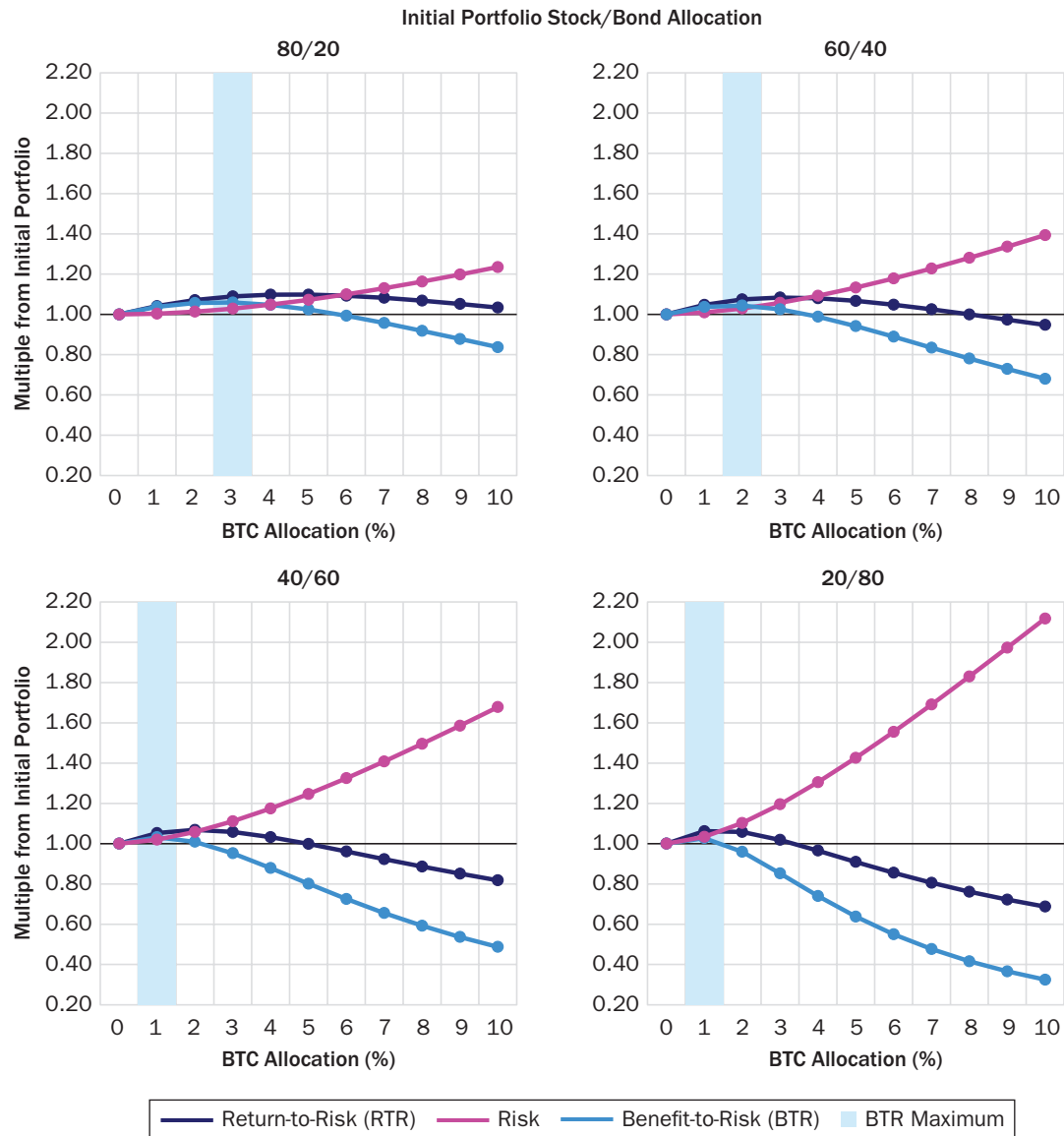


**SOURCE:** Bloomberg, Galaxy, MSCI, Invesco. Based on simulated returns generated using historical sample beginning 1/1/2014 through 12/31/2023.



### EXHIBIT 11

#### Simulated Multi-Asset Portfolios Outcomes—Assuming 50% of Historical BTC Return

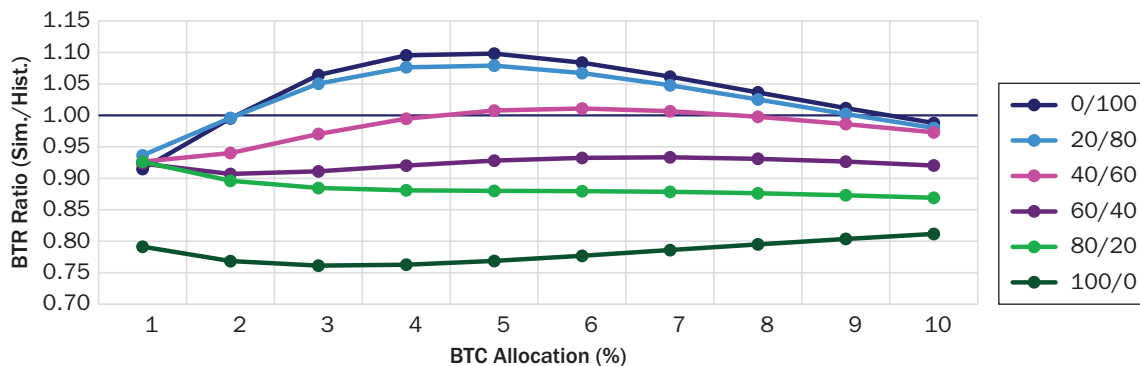


**NOTE:** Based on simulated returns generated using historical sample beginning 1/1/2014 through 12/31/2023.

**SOURCE:** Bloomberg, Galaxy, MSCI, Invesco.

## EXHIBIT 12

### Benefit-to-Risk Comparison (BTR ratio: simulated BTR/historical BTR)



**NOTE:** Based on historical returns for sample period beginning 1/1/2014 through 12/31/2023 and simulated returns generated using the same sample period.

**SOURCE:** Bloomberg, Galaxy, Invesco.

While it might have been expected that considering 10,000 alternative outcomes should result in different maxima from those seen in the historical analyses, the simulation analyses provide insights on three key points:

1. The benefits offered by small allocations to BTC are robust to the consideration of a nontrivial set of alternative BTC return outcomes.
2. Large allocations to BTC should be considered carefully as simulation results are indicative of lower benefits at higher allocations.
3. Diversification and rebalancing are critical for risk management and for extracting benefits from BTC allocations.

The simulation results effectively provide a robustness check for the historical analysis and are supportive of benefits that can accrue from small BTC allocations.

## CONCLUSION

In this article, we explored the implications for multi-asset investors of including an allocation to BTC in their portfolios. We began our analysis with reviewing the historical record to assess the observed characteristics of BTC. We then examined the historical implications of a broad range of stock/bond portfolios with various allocations to BTC, considering different return assumptions and highlighting the importance of position sizing and portfolio rebalancing. We also explored allocation implications across a wide range of simulated portfolio outcomes. Given the indeterminate nature of BTC returns, we intentionally avoided specialized optimization techniques or the consideration of alternative investor utility functions. That is noted only to indicate that the insights gained from this work are likely to be relevant to multi-asset investors, regardless of how they approach portfolio construction. From this analysis we extracted three key insights regarding allocations to BTC in multi-asset portfolios:

1. Small allocations to BTC appear to offer notable benefits to multi-asset portfolios—even when return expectations for BTC are significantly reduced.
2. Allocations to BTC should be evaluated relative to an investor's initial risk preferences.
3. BTC allocation sizing and portfolio rebalancing are critical aspects of risk management for portfolios with BTC allocations.

Insights two and three above hold regardless of the future path of BTC. Insight one requires additional context given the continuing evolution of the landscape for crypto assets. In considering the historical record, investors should understand that what has been experienced in the observable past in the crypto space is the introduction of a new set of financial instruments that hadn't previously existed. Not only are the instruments new, the markets supporting the purchase and sale of those instruments are also new and are evolving at the same time. Regulation is also a key concern. Regulations for crypto instruments are currently being introduced and are also likely to change as markets and regulators evaluate the impact of these instruments on financial markets and investors. Competition to BTC from alternative crypto instruments has also increased. All these elements, along with the changes affecting other parts of the capital markets, are likely to influence the return, risk, and correlation characteristics of BTC in the future.

Furthermore, any inferences derived from the historical record of BTC's relationship with other assets should be considered with care. We have provided evidence that price and volume dynamics appear to have changed notably relative to the time BTC first appeared. Not only have BTC's characteristics been changing as the instrument matures, but the limited history we have available includes a global pandemic, massive government intervention globally, significant increases in interest rates and inflation, and drastically varying stock–bond correlations. Ascertaining forward-looking asset relationships using returns from this period, even for traditional assets such as stocks and bonds, is likely to be problematic.

As is often noted in the financial services industry, past performance is no guarantee of future outcomes. This guidance is even more relevant for investors in crypto instruments, generally, and for BTC, in particular. That said, on the basis of the analysis presented and the assumption that future BTC returns will be in line with historical returns, BTC allocations appear to offer incremental benefits to multi-asset portfolios in excess of incremental risk for allocations of up to 5% for more aggressive portfolios and for allocations of up to 2% for more conservative portfolios. Assuming lower future returns for BTC equal to 50% of the historical return, we show benefits for BTC allocations of up to 3% for more aggressive portfolios and of no more than 1% for more conservative portfolios. Benefits at higher allocations are also possible with greater rebalancing frequencies. All BTC allocations, regardless of the potential for additional benefit and the rebalancing frequency used, should be weighed against the expected increases in risk that result from those allocations.

## APPENDIX

### AN OVERVIEW OF THE BENEFIT-TO-RISK METRIC

The starting point for assessing of the implications of BTC allocations to multi-asset portfolios is the investor's initial portfolio allocation. The initial portfolio contains information about an investor's initial risk preferences that are relevant to balancing the trade-off between the potential return benefits offered by BTC allocations against the increased risks that result from those allocations. This is aligned with the notion that aggressive (less risk-averse) investors are likely to be more willing to accept greater risk in pursuit of higher returns than conservative (more risk-averse) investors.

In considering the benefit-to-risk trade-off, we seek to consider the efficiency with which a portfolio can extract returns from the capital markets rather than strictly focusing on absolute return improvements. As such, we consider the benefits offered by BTC allocations in terms of RTR, or rather, return per unit of risk (See equation A1 below). A construct that should be familiar to most multi-asset investors.

Incremental benefits offered by increasing BTC allocations are assessed relative to the incremental risks that result from those allocations using RTR and risk multiples that relate these characteristics back to the investor's initial portfolio (see equations A2a and A2b below). Using multiples conveniently expresses benefits and risks in the same terms but also conveniently allows us to understand, in percentage terms relative to the initial portfolio, the incremental increases in both RTR and risk. The calculation of the Benefit-to-Risk (BTR) metric proceeds as follows.

We first calculate the RTR for a specific portfolio allocation ( $P_a$ ) under consideration as:

$$\text{Return-to-Risk (RTR)} = \frac{\text{Return}_{P_a}}{\text{Risk}_{P_a}} \quad (\text{A1})$$

Then the incremental portfolio allocation multiples are calculated relative to the initial portfolio ( $P_i$ ) as:

$$\text{RTR Multiple}_{P_a} = \frac{\text{RTR}_{P_a}}{\text{RTR}_{P_i}} \quad (\text{A2a})$$

and

$$\text{Risk Multiple}_{P_a} = \frac{\text{Risk}_{P_a}}{\text{Risk}_{P_i}} \quad (\text{A2b})$$

And, finally, the Benefit-to-Risk metric can be calculated as:

$$\text{Benefit-to-Risk}_{P_a} = \frac{\text{RTR Multiple}_{P_a}}{\text{Risk Multiple}_{P_a}} \quad (\text{A3})$$

The BTR metric is a conservative, relatively uncomplicated, and intuitive metric that allows for a clear understanding of the trade-off between the incremental increases in benefits to the incremental increases in risk. BTR values greater than one indicate that the incremental benefits offered by BTC are greater than the incremental risks. BTR values less than one indicate that incremental risks resulting from BTC are greater than incremental benefits. Ultimately, the use of this metric is aligned with the notion that investors should demand some type of benefit for accepting greater risk. BTC allocations with increasing benefits-to-risk should be considered carefully while BTC allocations with decreasing benefits-to-risk should likely be avoided.

**EXHIBIT A1**

**Historical Multi-Asset Portfolios Outcomes—Assuming 50% of Historical BTC Return**

		Portfolio Summary Statistics											
		BTC Allocation (%)											
		0	1	2	3	4	5	6	7	8	9	10	
Initial Portfolio Stock/Bond Allocation	100/0	Return (%)	7.41	8.14	8.82	9.47	10.08	10.67	11.22	11.75	12.25	12.73	13.19
		Risk (%)	14.48	14.53	14.70	14.95	15.28	15.65	16.06	16.50	16.97	17.45	17.94
		Return-to-Risk (RTR)	0.51	0.56	0.60	0.63	0.66	0.68	0.70	0.71	0.72	0.73	0.73
		Risk Multiple	1.00	1.00	1.02	1.03	1.05	1.08	1.11	1.14	1.17	1.20	1.24
		RTR Multiple	1.00	1.09	1.17	1.24	1.29	1.33	1.36	1.39	1.41	1.43	1.44
		Benefit-to-Risk	1.00	1.09	1.16	1.20	1.22	1.23	1.23	1.22	1.20	1.18	1.16
	80/20	Return (%)	6.42	7.18	7.89	8.57	9.21	9.82	10.39	10.94	11.47	11.97	12.45
		Risk (%)	11.47	11.57	11.82	12.18	12.62	13.11	13.64	14.19	14.77	15.35	15.95
		Return-to-Risk (RTR)	0.56	0.62	0.67	0.70	0.73	0.75	0.76	0.77	0.78	0.78	0.78
		Risk Multiple	1.00	1.01	1.03	1.06	1.10	1.14	1.19	1.24	1.29	1.34	1.39
		RTR Multiple	1.00	1.11	1.19	1.26	1.30	1.34	1.36	1.38	1.39	1.39	1.39
		Benefit-to-Risk	1.00	1.10	1.16	1.18	1.18	1.17	1.14	1.11	1.08	1.04	1.00
	60/40	Return (%)	5.40	6.19	6.94	7.65	8.31	8.95	9.55	10.12	10.67	11.19	11.69
		Risk (%)	8.68	8.84	9.20	9.70	10.28	10.91	11.57	12.25	12.93	13.62	14.31
		Return-to-Risk (RTR)	0.62	0.70	0.75	0.79	0.81	0.82	0.83	0.83	0.82	0.82	0.82
		Risk Multiple	1.00	1.02	1.06	1.12	1.18	1.26	1.33	1.41	1.49	1.57	1.65
		RTR Multiple	1.00	1.13	1.21	1.27	1.30	1.32	1.33	1.33	1.33	1.32	1.31
		Benefit-to-Risk	1.00	1.11	1.14	1.13	1.10	1.05	0.99	0.94	0.89	0.84	0.80
	40/60	Return (%)	4.35	5.18	5.96	6.70	7.39	8.05	8.68	9.28	9.85	10.39	10.91
		Risk (%)	6.21	6.45	6.96	7.63	8.38	9.17	9.96	10.76	11.56	12.34	13.12
		Return-to-Risk (RTR)	0.70	0.80	0.86	0.88	0.88	0.88	0.87	0.86	0.85	0.84	0.83
		Risk Multiple	1.00	1.04	1.12	1.23	1.35	1.48	1.60	1.73	1.86	1.99	2.11
		RTR Multiple	1.00	1.15	1.22	1.25	1.26	1.26	1.24	1.23	1.22	1.20	1.19
		Benefit-to-Risk	1.00	1.10	1.09	1.02	0.93	0.85	0.78	0.71	0.65	0.61	0.56
20/80	Return (%)	3.26	4.14	4.95	5.72	6.45	7.14	7.79	8.41	9.00	9.57	10.11	
	Risk (%)	4.44	4.76	5.44	6.28	7.18	8.09	9.00	9.88	10.75	11.59	12.42	
	Return-to-Risk (RTR)	0.73	0.87	0.91	0.91	0.90	0.88	0.87	0.85	0.84	0.83	0.81	
	Risk Multiple	1.00	1.07	1.22	1.41	1.62	1.82	2.02	2.22	2.42	2.61	2.79	
	RTR Multiple	1.00	1.18	1.24	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.11	
	Benefit-to-Risk	1.00	1.10	1.01	0.88	0.76	0.66	0.58	0.52	0.47	0.43	0.40	
0/100	Return (%)	2.15	3.06	3.92	4.72	5.48	6.20	6.88	7.53	8.14	8.73	9.29	
	Risk (%)	4.19	4.48	5.17	6.03	6.96	7.89	8.81	9.71	10.58	11.44	12.26	
	Return-to-Risk (RTR)	0.51	0.68	0.76	0.78	0.79	0.79	0.78	0.78	0.77	0.76	0.76	
	Risk Multiple	1.00	1.07	1.23	1.44	1.66	1.88	2.10	2.32	2.53	2.73	2.93	
	RTR Multiple	1.00	1.33	1.48	1.53	1.54	1.53	1.52	1.51	1.50	1.49	1.48	
	Benefit-to-Risk	1.00	1.25	1.20	1.06	0.92	0.81	0.72	0.65	0.59	0.54	0.50	

SOURCE: Bloomberg, Galaxy, MSCI, Invesco. Based on historical returns beginning 1/1/2014 through 12/31/2023.

**EXHIBIT A2**

**Simulated Multi-Asset Portfolios Outcomes—Assuming Historical BTC Return**

		Portfolio Summary Statistics											
		BTC Allocation (%)											
		0	1	2	3	4	5	6	7	8	9	10	
Initial Portfolio Stock/Bond Allocation	100/0	Return-to-Risk (RTR)	0.72	0.67	0.73	0.78	0.83	0.87	0.91	0.94	0.97	1.00	1.03
		Risk Multiple	1.00	1.00	1.01	1.02	1.03	1.05	1.07	1.10	1.13	1.15	1.18
		RTR Multiple	1.00	0.93	1.01	1.08	1.14	1.20	1.26	1.31	1.35	1.39	1.43
		Benefit-to-Risk	1.00	0.93	1.00	1.06	1.11	1.14	1.17	1.19	1.20	1.20	1.20
	80/20	Return-to-Risk (RTR)	0.74	0.82	0.89	0.94	0.97	1.00	1.02	1.04	1.05	1.06	1.06
		Risk Multiple	1.00	1.01	1.02	1.04	1.07	1.10	1.13	1.17	1.21	1.25	1.30
		RTR Multiple	1.00	1.11	1.19	1.26	1.31	1.35	1.38	1.40	1.41	1.42	1.43
		Benefit-to-Risk	1.00	1.10	1.17	1.21	1.23	1.23	1.22	1.20	1.17	1.14	1.10
	60/40	Return-to-Risk (RTR)	0.78	0.88	0.95	0.99	1.02	1.04	1.05	1.06	1.06	1.06	1.06
		Risk Multiple	1.00	1.01	1.04	1.08	1.12	1.17	1.23	1.29	1.35	1.42	1.49
		RTR Multiple	1.00	1.13	1.22	1.28	1.32	1.34	1.35	1.36	1.36	1.36	1.36
		Benefit-to-Risk	1.00	1.11	1.17	1.19	1.17	1.14	1.10	1.06	1.01	0.96	0.91
	40/60	Return-to-Risk (RTR)	0.83	0.96	1.03	1.06	1.08	1.08	1.07	1.06	1.05	1.04	1.03
		Risk Multiple	1.00	1.03	1.08	1.15	1.23	1.32	1.41	1.51	1.61	1.72	1.83
		RTR Multiple	1.00	1.15	1.23	1.27	1.29	1.29	1.29	1.27	1.26	1.25	1.24
		Benefit-to-Risk	1.00	1.12	1.14	1.11	1.05	0.98	0.91	0.84	0.78	0.73	0.68
	20/80	Return-to-Risk (RTR)	0.86	1.01	1.08	1.09	1.08	1.06	1.04	1.02	1.00	0.99	0.97
		Risk Multiple	1.00	1.05	1.14	1.25	1.39	1.54	1.69	1.85	2.02	2.18	2.35
		RTR Multiple	1.00	1.18	1.25	1.27	1.26	1.24	1.21	1.19	1.17	1.15	1.14
		Benefit-to-Risk	1.00	1.13	1.10	1.01	0.90	0.80	0.72	0.64	0.58	0.53	0.48
0/100	Return-to-Risk (RTR)	0.56	0.76	0.86	0.90	0.91	0.91	0.90	0.89	0.89	0.88	0.87	
	Risk Multiple	1.00	1.03	1.12	1.24	1.39	1.55	1.72	1.89	2.07	2.25	2.43	
	RTR Multiple	1.00	1.34	1.52	1.59	1.62	1.62	1.60	1.59	1.57	1.56	1.55	
	Benefit-to-Risk	1.00	1.30	1.36	1.28	1.16	1.04	0.93	0.84	0.76	0.69	0.64	

**SOURCE:** Bloomberg, Galaxy, MSCI, Invesco. Based on simulated returns generated using historical sample beginning 1/1/2014 through 12/31/2023.



### EXHIBIT A3

#### Simulated Multi-Asset Portfolios Outcomes—Assuming 50% of Historical BTC Return

		Portfolio Summary Statistics											
		BTC Allocation (%)											
		0	1	2	3	4	5	6	7	8	9	10	
Initial Portfolio Stock/Bond Allocation	100/0	Return-to-Risk (RTR)	0.72	0.64	0.66	0.67	0.69	0.70	0.71	0.72	0.73	0.73	0.74
		Risk Multiple	1.00	1.00	1.00	1.01	1.02	1.04	1.05	1.07	1.09	1.12	1.14
		RTR Multiple	1.00	0.88	0.91	0.93	0.95	0.97	0.99	1.00	1.01	1.02	1.02
		Benefit-to-Risk	1.00	0.88	0.90	0.92	0.93	0.94	0.94	0.93	0.92	0.91	0.90
	80/20	Return-to-Risk (RTR)	0.74	0.77	0.80	0.81	0.82	0.82	0.81	0.80	0.79	0.78	0.77
		Risk Multiple	1.00	1.00	1.01	1.03	1.05	1.07	1.10	1.13	1.16	1.20	1.24
		RTR Multiple	1.00	1.04	1.07	1.09	1.10	1.10	1.09	1.08	1.07	1.05	1.03
		Benefit-to-Risk	1.00	1.04	1.06	1.06	1.05	1.02	0.99	0.96	0.92	0.88	0.84
	60/40	Return-to-Risk (RTR)	0.78	0.82	0.84	0.84	0.84	0.83	0.82	0.80	0.78	0.76	0.74
		Risk Multiple	1.00	1.01	1.03	1.06	1.09	1.13	1.18	1.23	1.28	1.34	1.39
		RTR Multiple	1.00	1.05	1.07	1.08	1.08	1.07	1.05	1.03	1.00	0.97	0.95
		Benefit-to-Risk	1.00	1.04	1.04	1.03	0.99	0.94	0.89	0.83	0.78	0.73	0.68
	40/60	Return-to-Risk (RTR)	0.84	0.88	0.89	0.88	0.86	0.84	0.80	0.77	0.74	0.71	0.68
		Risk Multiple	1.00	1.02	1.06	1.11	1.17	1.25	1.33	1.41	1.50	1.59	1.68
		RTR Multiple	1.00	1.05	1.07	1.06	1.03	1.00	0.96	0.92	0.89	0.85	0.82
		Benefit-to-Risk	1.00	1.03	1.01	0.95	0.88	0.80	0.73	0.66	0.59	0.54	0.49
	20/80	Return-to-Risk (RTR)	0.86	0.91	0.91	0.88	0.83	0.78	0.74	0.69	0.65	0.62	0.59
		Risk Multiple	1.00	1.03	1.10	1.20	1.30	1.43	1.56	1.69	1.83	1.97	2.12
		RTR Multiple	1.00	1.06	1.06	1.02	0.97	0.91	0.86	0.81	0.76	0.72	0.69
		Benefit-to-Risk	1.00	1.03	0.96	0.85	0.74	0.64	0.55	0.48	0.42	0.37	0.32
0/100	Return-to-Risk (RTR)	0.56	0.64	0.67	0.66	0.63	0.60	0.57	0.53	0.51	0.48	0.46	
	Risk Multiple	1.00	1.02	1.09	1.18	1.30	1.43	1.57	1.71	1.86	2.02	2.18	
	RTR Multiple	1.00	1.14	1.18	1.17	1.12	1.06	1.00	0.95	0.90	0.86	0.82	
	Benefit-to-Risk	1.00	1.12	1.09	0.99	0.86	0.74	0.64	0.55	0.48	0.42	0.38	

**SOURCE:** Bloomberg, Galaxy, MSCI, Invesco. Based on simulated returns generated using historical sample beginning 1/1/2014 through 12/31/2023.

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